Local Reasoning about Storable Locks and Threads

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Shared variable concurrent programs

- **Dijkstra**
  - Programs should be insensitive to relative execution speeds

- **Brinch Hansen / Hoare**
  - Shared variables should be encapsulated and their access controlled
  - Monitors
    - Compiler could check if encapsulation violated for variables
    - Solo operating system written almost entirely with safe primitives
    - But what about the heap? Needed for multi-user OSes

- **Owicki & Gries / Jones**
  - Limit interference through shared state with predicates / relations

- **O’Hearn**
  - Concurrent separation logic: encapsulation checking for the heap
    - “Size” of shared state can change
    - “Topology” of access control still fixed
typedef struct NODE {
    int Val;
    struct NODE* Next;
} NODE;

LOCK lock;
NODE* head;

locate_coarse(int e) {
    NODE *prev, *curr;
    acquire(lock);
    prev = head;
    curr = prev->Next;
    while (curr->Val < e) {
        prev = curr;
        curr = prev->Next;
    }
    return (prev, curr);
}

typedef struct NODE {
    LOCK Lock;
    int Val;
    struct NODE* Next;
} NODE;

NODE* head;

locate_hand_over_hand(int e) {
    NODE *prev, *curr;
    prev = head;
    acquire(prev);
    curr = prev->Next;
    acquire(curr);
    while (curr->Val < e) {
        release(prev);
        prev = curr;
        curr = prev->Next;
        acquire(curr);
    }
    return (prev, curr);
}
Resources & CCRs vs locks

Optimistic / Idealistic

- syntactically determined critical regions

(resource r

with r do

: od)

(More) Realistic

- semantically determined critical regions

if (flag > 0) {
  acquire(l);
}
...

if (flag > 0) {
  release(l);
}
Locks on the stack vs locks in the heap

Optimistic / Idealistic

```
resource r

with r do
 :
 od
```

- bounded numbers of resources

(More) Realistic

```
l = new LOCK;
:
init(l);
:
acquire(l);
:
release(l);
:
finalize(l);
:
delete l;
```

- unbounded numbers of locks
Parallel composition vs dynamic thread creation

Optimistic / Idealistic

\[(\text{while } b \text{ do } (P_1 \parallel P_2)) \parallel P_3\]

(More) Realistic

\[\text{for } (i = 0; i < n; i++) \{\]
\[\quad t[i] = \text{fork(proc, i);}\]
\[\}\]
\[\vdots\]
\[\text{for } (i = 0; i < n; i++) \{\]
\[\quad \text{join(t[i]);}\]
\[\}\]

- bounded numbers of processes
- unbounded numbers of threads
Objective

• Program logics for analysis and verification of multithreaded heap-manipulating programs

• Goal: ease static access control
  – Allow unboundedly-many locks and threads
  – That live in the heap (to exploit indirection)

• but also aim to:
  – Retain local reasoning
  – Enable automation in program analysis
  – Treat more realistic programming language constructs
• Logic for storable locks and threads
  – Local reasoning preserved
  – Storable locks as resources

• Not only technical difficulties:
  – Storable locks “make theoreticians wince” (Richard Bornat)
  – Russell’s paradox is lurking nearby:
    heaps → locks → resource invariants → heaps
  – Analogous to stored procedures: Landin’s “knots in the store”
• First one top-level parallel composition: $C_1 \parallel \cdots \parallel C_n$
• Then dynamic thread creation

• Simplification: no shared mutable variables
  – shared mutable heap
  – global pre-initialized constants
  – local variables of threads

• General cases and details:
Concurrent separation logic [O’Hearn04]

- A Floyd/Hoare-style program logic
- Assertion language: $\star$ splits the state into disjoint parts
- Proof system:

  $$\begin{align*}
  & \{P\} \quad C \quad \{Q\} \\
  \xrightarrow{P*R} & \quad \{P*R\} \quad C \quad \{Q*R\} \\
  \end{align*}$$

  $$\begin{align*}
  & \{P_1\} \quad C_1 \quad \{Q_1\} \quad \{P_2\} \quad C_2 \quad \{Q_2\} \\
  \xrightarrow{P_1*P_2} & \quad \{P_1*P_2\} \quad C_1 \parallel C_2 \quad \{Q_1*Q_2\} \\
  \end{align*}$$

- Allows for local reasoning
- Processes access shared resources
- Synchronization via conditional critical regions:

  $$\text{with } r \text{ when } b \text{ do } C$$

  to be replaced
• Program state partitioned into (disjoint) substates owned by the different processes and locks
• Processes may access only parts of the state that they own
• Process interaction mediated using resource invariants
• Key in achieving local reasoning:
  – reasoning about each process in isolation
  – using the sequential semantics
locate_coarse(int e) {
    NODE *prev, *curr;
    acquire(lock);
    "have (exclusive access to) head list"
    prev = head;
    "head has a Next"
    curr = prev->Next;
    "curr has a Val"
    while (curr->Val < e) {
        prev = curr;
        "curr has a Next"
        curr = prev->Next;
    }
    return (prev, curr);
}

locate_hand_over_hand(int e) {
    NODE *prev, *curr;
    prev = head;
    acquire(prev);
    "have (exclusive access to) prev node"
    curr = prev->Next;
    "curr has a Lock"
    acquire(curr);
    "have curr node"
    while (curr->Val < e) {
        "prev is locked by this thread"
        release(prev);
        "don’t have prev node any more"
        prev = curr;
        curr = prev->Next;
        "curr has a Lock"
        acquire(curr);
        "have curr node"
    }
    return (prev, curr);
}

Need to know this even without owning curr node:
So ownership of a node comes with knowledge that the Next node has a Lock
Approach

• Lock → resource invariant
  – lock → sort $A(\cdot, \cdot)$
  – sort $A(\cdot, \cdot) \rightarrow$ resource invariant $I_{A}(\cdot, \cdot)$
  – first parameter – address of the lock

• Example:

```c
struct R {
    LOCK Lock;
    int Data;
};

I_{R}(l, v) \triangleq l:Data \rightarrow v
```

• Knots in the store cut by indirection through $A(\cdot, \cdot)$
• **Handles:** \( A(E, \vec{F}) \)
  
  – ensures that the lock at the address \( E \) exists and has the sort \( A \) and parameters \( \vec{F} \)
  
  – gives permission to acquire the lock
  
  – can be split among threads:
    
    • \( 1A(E, \vec{F}) = \frac{1}{2}A(E, \vec{F}) \ast \frac{1}{2}A(E, \vec{F}) \)
    
    • \(< 1\) – can acquire the lock
    
    • \(= 1\) – can finalize the lock

• **Locked-facts:** \( \text{Locked}_A(E, \vec{F}) \)

  – lock \( E \) is held by the thread owning \( \text{Locked}_A(E, \vec{F}) \)

  – ensures the existence of the lock
\[
\begin{align*}
\{ E \rightarrow \_ \} & \quad \text{init}_{A,F}(E) \quad \{ A(E, \vec{F}) \ast \text{Locked}_A(E, \vec{F}) \} \\
\{ A(E, \vec{F}) \ast \text{Locked}_A(E, \vec{F}) \} & \quad \text{finalize}(E) \quad \{ E \rightarrow \_ \} \\
\{ \text{Locked}_A(E, \vec{F}) \ast I_A(E, \vec{F}) \} & \quad \text{release}(E) \quad \{ \text{emp}_h \} \\
\{ \pi A(E, \vec{F}) \} & \quad \text{acquire}(E) \quad \{ \pi A(E, \vec{F}) \ast \text{Locked}_A(E, \vec{F}) \ast I_A(E, \vec{F}) \}
\end{align*}
\]
A simple example

```c
struct R {
    LOCK Lock;
    int Data;
} *x;

// $I_R(l) \triangleq l::Data \rightarrow$

initialize() {
    {emp\_h}
    x = new R;
    {x\rightarrow _* x::Data \rightarrow _} init\_R(x);
    {x::Data \rightarrow _* R(x)
        * Locked\_R(x)}
    x->Data = 0;
    {x::Data \rightarrow 0 * R(x)
        * Locked\_R(x)}
    release(x);
    {R(x)}
}

thread() {
    {1/2 R(x)}
    acquire(x);
    {x::Data \rightarrow _* 1/2 R(x)
        * Locked\_R(x)}
    x->Data++;
    {x::Data \rightarrow _* 1/2 R(x)
        * Locked\_R(x)}
    release(x);
    {1/2 R(x)}
}

cleanup() {
    {R(x)}
    acquire(x);
    {x::Data \rightarrow _* R(x)
        * Locked\_R(x)}
    finalize(x);
    {x\rightarrow _* x::Data \rightarrow _}
    delete x;
    {emp\_h}
}
```

Josh Berdine — Local Reasoning about Storable Locks
Assertion language model

- Semantic domains:
  Stacks = Vars →_{\text{fin}} Values
  Heaps = Locations →_{\text{fin}}
  \((\text{Cell(Values)} \cup \text{Lock(Sorts × LockValues × LockPerms)})\)

- each program proof associates each sort with an invariant:
  \(I_A(\vec{E}) : \text{Sorts} \rightarrow \text{Values}^+ \rightarrow \mathcal{P}(\text{Stacks} \times \text{Heaps})\)

- Satisfaction relation: \((s, h) \models_k \Phi\)
  \((s, h) \models_k E \rightarrow_{F} \Leftrightarrow h = [[E]_s : \text{Cell}([[F]_s])]\)
  \((s, h) \models_k \pi A(E) \Leftrightarrow h = [[E]_s : \text{Lock}(A, U, [[\pi]_s]) \land [[\pi]_s > 0\)
  \((s, h) \models_k \text{Locked}_A(E) \Leftrightarrow h = [[E]_s : \text{Lock}(A, k, 0)]\)

* adds up permissions for locks and their values:
  \(U \ast k = k, \ U \ast U = U, \ k \ast j \text{ undefined}\)
Semantics of programs

- \( pc \in \{1, \ldots, n\} \rightarrow \text{ProgPoint} \)
- \( F \subseteq \text{ProgPoint} \times \text{Command} \times \text{ProgPoint} \)
- \( \rightarrow_s \) is the least relation satisfying:

\[
\frac{(v, C, v') \in F \quad k \in \{1, \ldots, n\} 
\quad C, (s, h) \rightsquigarrow_k q
}{pc[k : v], (s, h) \rightarrow_s pc[k : v'], q}
\]

\[
x = E, (s[x : (u, 1)], h) \quad \rightsquigarrow_k (s[x : ([E]_s[x:(u,1)], 1)], h)
\]

\[
x = [E], (s[x : (u, 1)], h[e : \text{Cell}(u)]) \quad \rightsquigarrow_k (s[x : (u, 1)], h[e : \text{Cell}(u)]), e = [E]_s[x:(u,1)]
\]

\[
[E] = F, (s, h[[E]_s : \text{Cell}(u)]) \quad \rightsquigarrow_k (s, h[[E]_s : \text{Cell}([F]_s)])
\]

\[
x = \text{new}, (s[x : (u, 1)], h) \quad \rightsquigarrow_k (s[x : (v, 1)], h[v : \text{Cell}(w)]), \text{if } h(v)\uparrow
\]

\[
delete E, (s, h[[E]_s : \text{Cell}(u)]) \quad \rightsquigarrow_k (s, h)
\]

\[
\text{init}_A(E), (s, h[[E]_s : \text{Cell}(u)]) \quad \rightsquigarrow_k (s, h[[E]_s : \text{Lock}(A, k, 1)])
\]

\[
\text{finalize}(E), (s, h[[E]_s : \text{Lock}(A, k, 1)]) \quad \rightsquigarrow_k (s, h[[E]_s : \text{Cell}(u)])
\]

\[
\text{assume}(G'), (s, h) \quad \rightsquigarrow_k (s, h), \text{if } [G']_s = \text{true}
\]

\[
\text{assume}(G), (s, h) \quad \rightsquigarrow_k (s, h), \text{if } [G']_s = \text{false}
\]

\[
\text{acquire}(E), (s, h[[E]_s : \text{Lock}(A, 0, \pi)]) \quad \rightsquigarrow_k (s, h[[E]_s : \text{Lock}(A, k, \pi)])
\]

\[
\text{acquire}(E), (s, h[[E]_s : \text{Lock}(A, j, \pi)]) \quad \rightsquigarrow_k \quad \text{if } j > 0
\]

\[
\text{release}(E), (s, h[[E]_s : \text{Lock}(A, k, \pi)]) \quad \rightsquigarrow_k (s, h[[E]_s : \text{Lock}(A, 0, \pi)])
\]
Flies in the ointment

- Consider invariants:
  \[ I_A(x, y) \triangleq B(y, x) \quad I_B(x, y) \triangleq A(y, x) \]

- with code:
  ```
  \{ x \mapsto _ * y \mapsto _ \} \\
  \text{init}_{A,y}(x); \\
  \text{init}_{B,x}(y); \\
  \{ A(x, y) * \text{Locked}_A(x, y) * B(y, x) * \text{Locked}_B(y, x) \} \\
  \text{release}(x); \\
  \{ A(x, y) * \text{Locked}_B(y, x) \} \\
  \text{release}(y); \\
  \{ \text{emp}_h \} \\
  ```

- Postcondition has forgotten that locks \( x \) and \( y \) exist!
- Logic may not detect a memory leak
- Formulating soundness becomes non-trivial
Soundness (cheating version)

- Usual interleaving-based operational semantics
- Program $C_1 \parallel \cdots \parallel C_n$
- $\vdash \{P_k\} C_k \{Q_k\}$
- Resource invariants are precise
  - Unambiguously pick out an area of the heap
- Theorem:
  \[
  \left[ \Phi \right]^k = \{(s, h) : (s, h) \models_k \Phi \}
  \]
  \[
  \text{If } \sigma_0 \in \left( \bigodot_{k=1}^n \left[ P_k \right]^k \right) \ast \left( \bigodot \{\text{invariants for free locks in } \sigma_0\} \right),
  \]
  \[
  \text{then the program is “safe”}
  \]
  \[
  \text{and } \sigma_f \in \left( \bigodot_{k=1}^n \left[ Q_k \right]^k \right) \ast \left( \bigodot \{\text{invariants for free locks in } \sigma_f\} \right)
  \]
- Cheat: statement about $\sigma_0/\sigma_f$ uses information about free locks in $\sigma_0/\sigma_f$
Closure

• How can we find all free locks allocated in a state from a set $p$?
  – Take $\sigma \in p$
  – Conjoin to $\sigma$ resource invariants for all locks with value $U$ in $\sigma$
  – and set the value of these locks to 0
  – Do the same for every state obtained in this way...

• Definition:
  The resulting states without locks with value $U$ form the closure of $p$: $\langle p \rangle$

• Example: $\langle R(x) \rangle$ where $I_R(l) = (l:\text{Data}\to-)$
• Example: $\langle B(y,x) \rangle$ where $I_B(x,y) = A(y,x)$ and $I_A(x,y) = B(y,x)$
• Are we guaranteed to add invariants for all free locks in this way?
• No! – Due to self-contained sets of locks
Admissibility of resource invariants

• Admissibility disallows self-contained sets of locks
• If resource invariants are admissible, closure finds all free locks

• Definition:

  Resource invariants for lock sorts \( L \) are admissible if there do not exist:
  – a non-empty set \( L \) of lock sorts from \( L \) with parameters
  – a state \( \sigma \in \{ \text{invariants for all locks in } L \} \)
  such that the permission associated with the every lock from \( L \) in \( \sigma \) is 1

• Examples:
  – \( \{ I_R(l) \triangleq l:\text{Data} \rightarrow - \} \) is admissible
  – \( \{ I_A(x, y) \triangleq B(y, x), \ I_B(x, y) \triangleq A(y, x) \} \) is not
Soundness

- Usual interleaving-based operational semantics
- Program $C_1 \parallel \cdots \parallel C_n$
- $\vdash \{P_k\} C_k \{Q_k\}$
- Resource invariants are precise

- Theorem:
  Suppose that
  
  - either resource invariants are admissible
  - or one of $Q_k$ is intuitionistic (does not notice heap extension)

  If $\sigma_0 \in \left\langle \bigotimes_{k=1}^{n} \left[ P_k \right]^k \right\rangle$, then the program is “safe”

  and $\sigma_f \in \left\langle \bigotimes_{k=1}^{n} \left[ Q_k \right]^k \right\rangle$
Dynamic thread creation

- Programs: \( \text{let } f_1() = C_1, \ldots, f_n() = C_n \text{ in } C \)

- Two new commands: \( x = \text{fork}(f) \) and \( \text{join}(E) \)

- Assertion language: thread handles \( \text{tid}_f(E) \)
  - thread running \( f \) with identifier \( E \) exists
  - gives permission to join it
  - only one thread can join any given thread

- Satisfaction relation: \( (s, h, t) \models_k \Phi \)
  - \( t \) – thread pool
Axioms for fork and join

- Need to give up the precondition of the thread at fork:

$$\Gamma, \{P\} f() \{Q\} \vdash \{P\} x = \text{fork}(f) \{\text{emp}_h \land \text{tid}_f(x)\}$$

- and receive the postcondition at join:

$$\Gamma, \{P\} f() \{Q\} \vdash \{\text{emp}_h \land \text{tid}_f(E)\} \text{join}(E) \{Q\}$$

where $\text{fv}(\{P, Q\}) \subseteq \text{GlobalConsts}$

- Other axioms adjusted accordingly
Soundness

• Proof of the program let $f_1() = C_1, \ldots, f_n() = C_n$ in $C$:

\[
\begin{align*}
\Gamma & \vdash \{P_1\} C_1 \{Q_1\} \\
\vdots \\
\Gamma & \vdash \{P_n\} C_n \{Q_n\} \\
\Gamma & \vdash \{P\} C \{Q\}
\end{align*}
\]

where

\[
\Gamma = \{P_1\} f() \{Q_1\}, \ldots, \{P_n\} f() \{Q_n\}
\]

• Technical issues:

  – Soundness conditions:
    • $P_k$ are precise
    • $P_k$ and $Q_k$ have an empty lockset (no lock in a state satisfying them has a value other than U)

  – Same circularity problem as with locks: $\text{tid}_f \rightarrow Q_f \rightarrow \text{tid}_f$

  – Admissibility, closure, and soundness can be generalized
Compared to concurrent separation logic

- Original concurrent separation logic can reason about storable locks:
  - represent them as cells storing the identifier of the thread owning the lock
  - build a global invariant of memory as a whole

- Drawbacks:
  - lots of auxiliary state ⇒ horrible proofs
  - reasoning is not modular
  - automation is infeasible
Compared to RGSep [Vafeiadis+07]

- **RGSep** – Combination of Jones’ rely-guarantee and separation logic
  - Locks not treated natively
  - Uses rely-guarantee to simplify reasoning about the global invariant
  - (+) Reasoning about complex finely-grained concurrency algorithms
  - (−) Awkward reasoning about programs that allocate and deallocate many simple data structures

- One fancy pre-allocated data structure vs many dynamically allocated simpler ones

- We’d like both at once
Summary

• Proposed a Floyd/Hoare-style program logic for
  – concurrent, heap-manipulating programs that:
  – allows local reasoning about unboundedly-many storable locks and threads
    • i.e., more realistic concurrent programming primitives
  – is strong enough to prove some examples published as challenges
    • piece of multicasting code
    • lock-coupling list operations
  – is set up to found a program analysis
    • thread-local fixed-point semantics is an analysis scheme
  – is sound via a reasonably lightweight mechanism for cutting recursive knots in the heap
    • using only a simple semantics

• Want a semantic analysis of admissibility of resource invariants