

# Consumption Risksharing in Social Networks

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PRELIMINARY

# Introduction

- In many societies, formal insurance markets are missing or imperfect  
⇒ agents use informal arrangements
  - Such arrangements often take place in the social network.
- This paper builds a model of informal insurance in the social network.
  - Connections generate value which is used as *social collateral* to enforce informal contracts.
- Our model yields results consistent with stylized facts about limited risksharing in developing countries.

# Plan of the Talk

1. Theory: Informal insurance in social networks.

2. Model analysis:

A) The limits to risksharing;

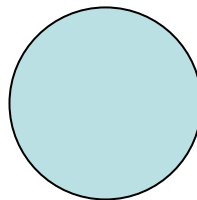
B) Constrained efficient arrangements and local sharing.

## Related literature

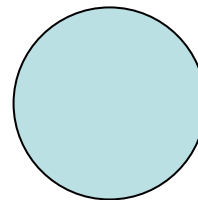
- Bloch, Genicot and Ray (2006): Characterize networks that are stable under certain risksharing arrangements.
- Bramoulle and Kranton (2005): Explore risksharing without enforcement constraints.
- Mobius and Szeidl (2006): Borrowing in a social network.
- Wilson (1968): Risksharing in syndicates.
- Cochrane (1991), Townsend (1994), Udry (1994): Limits to full risk-sharing in the data.

# The benefits of risksharing

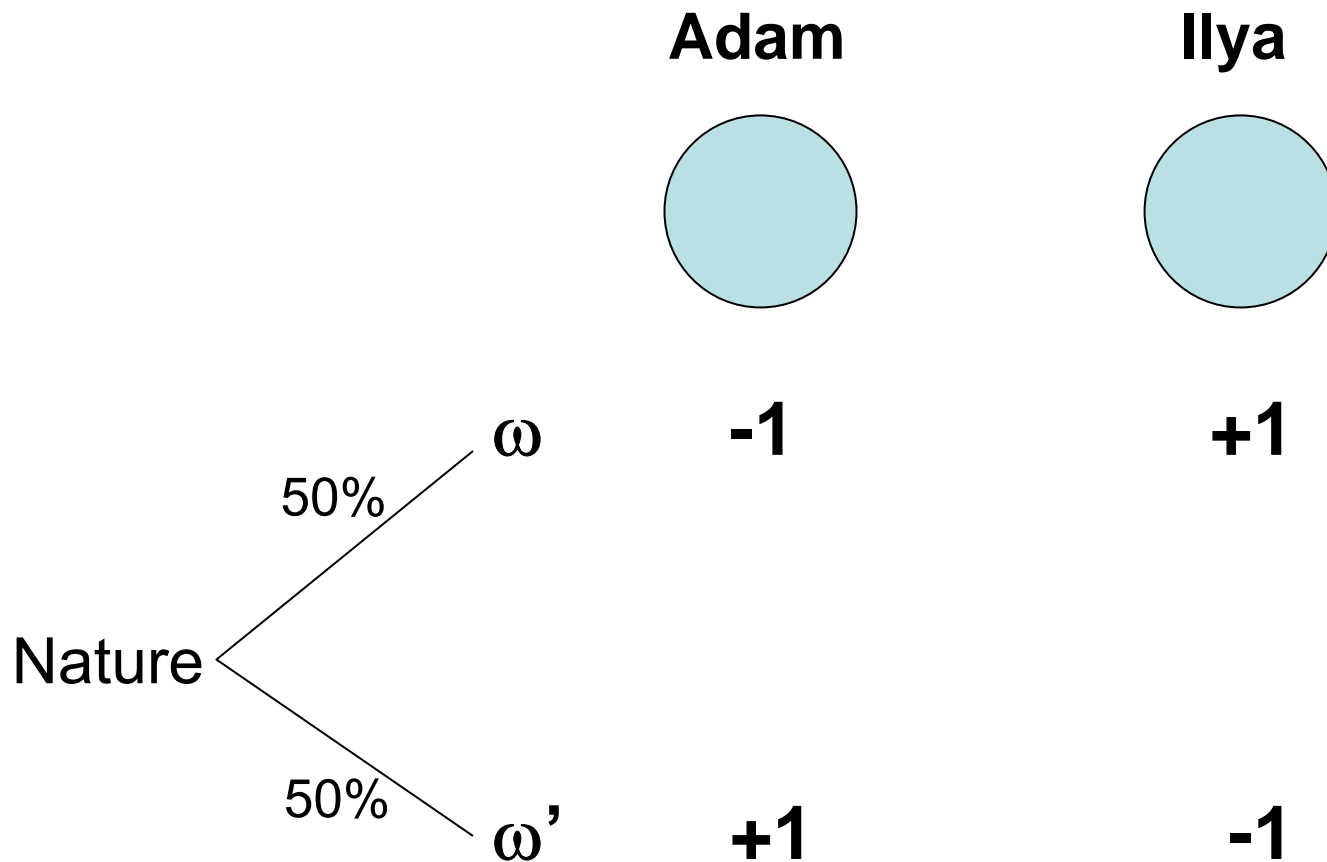
**Adam**



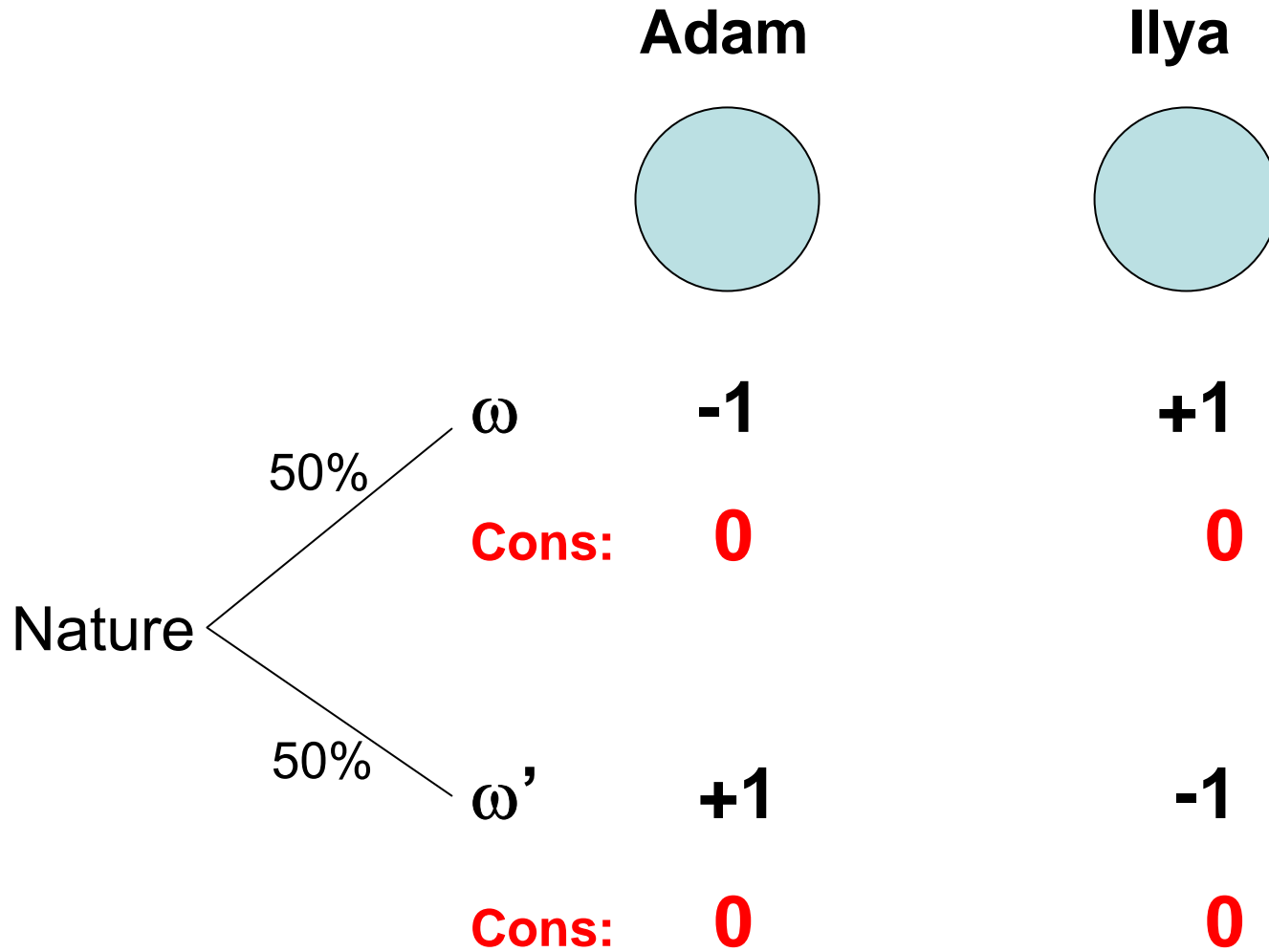
**Ilya**



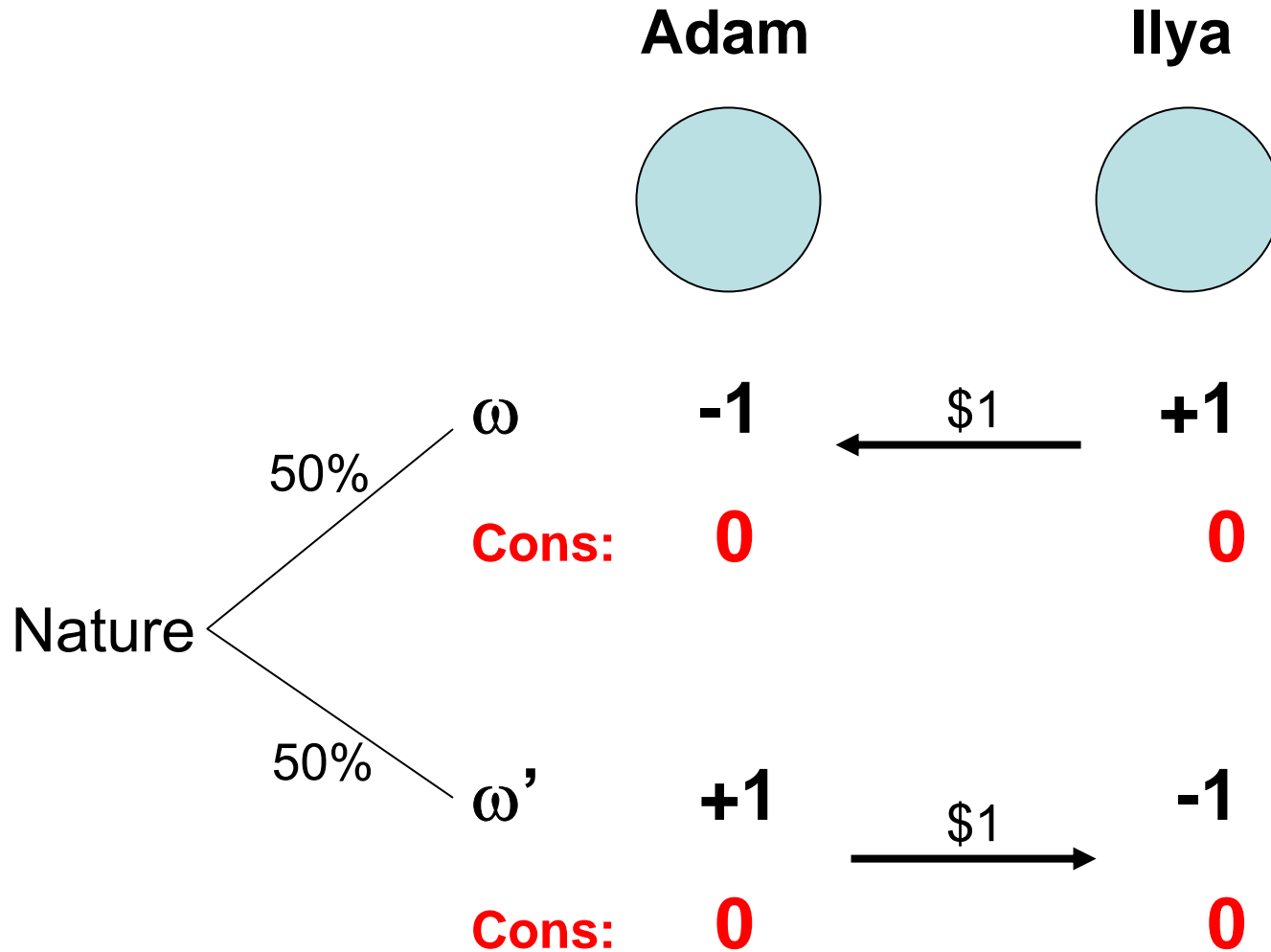
# The benefits of risksharing



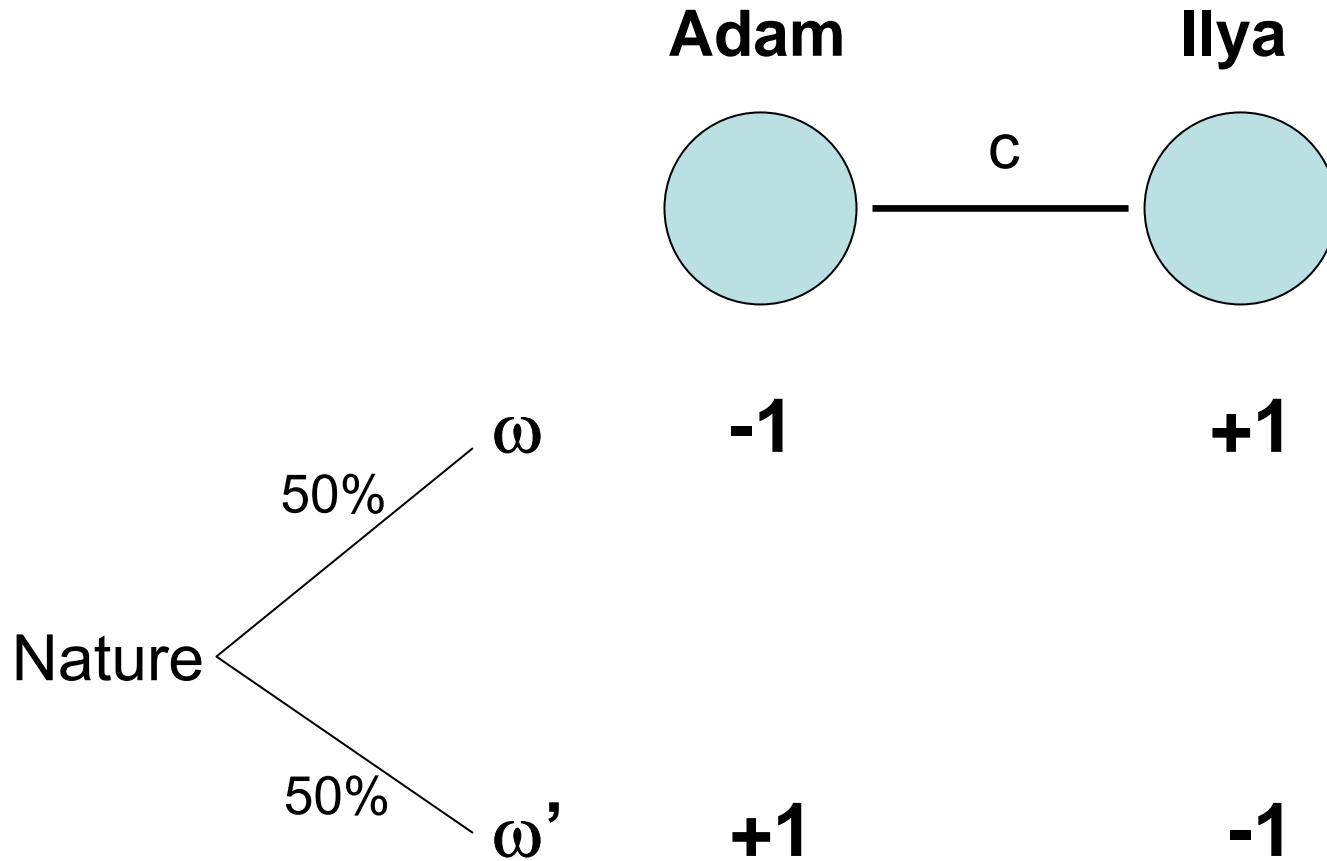
# The benefits of risksharing



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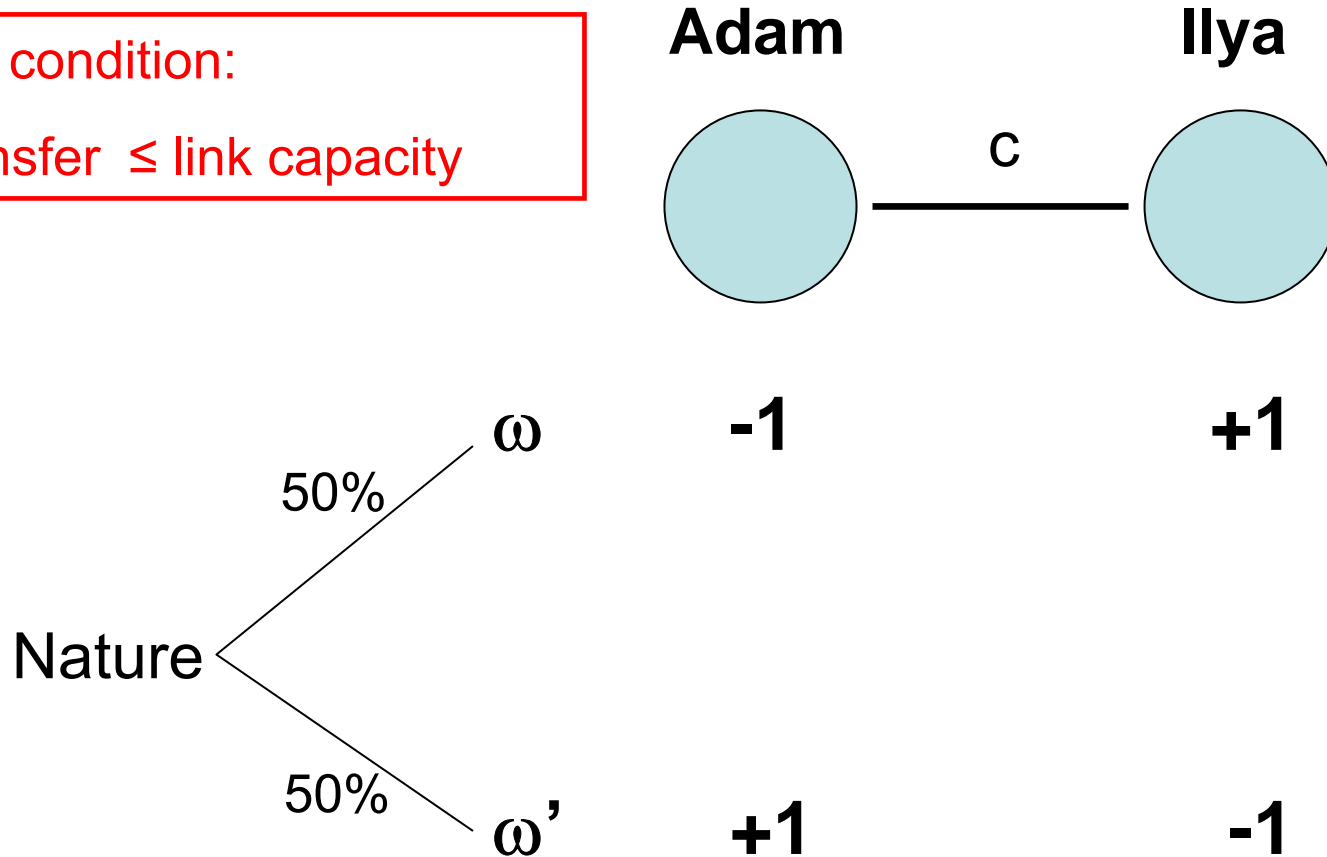
# Risksharing with informal enforcement



# Risksharing with informal enforcement

Key condition:

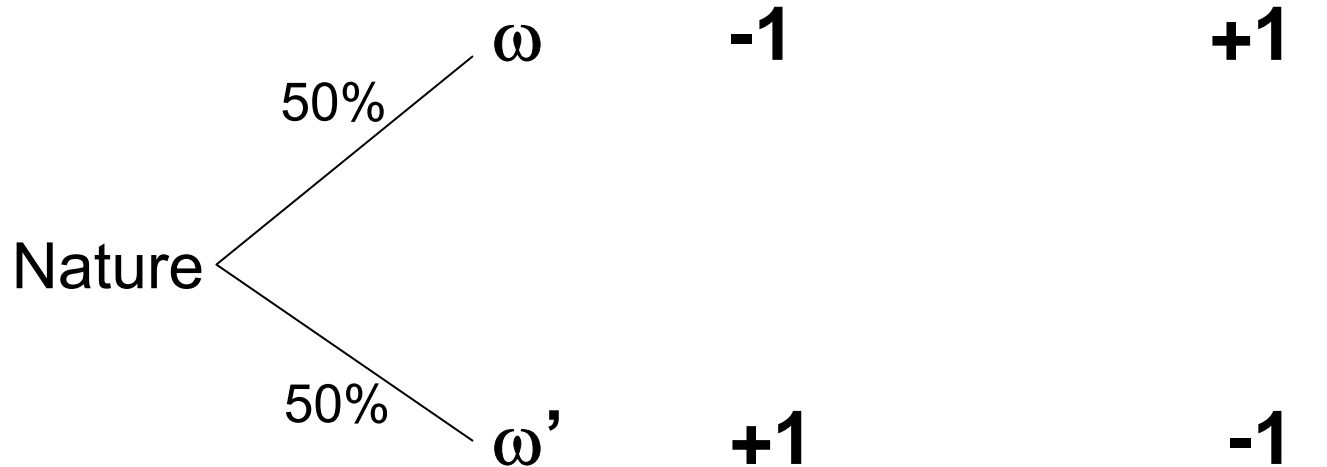
Transfer  $\leq$  link capacity



# Risksharing with informal enforcement

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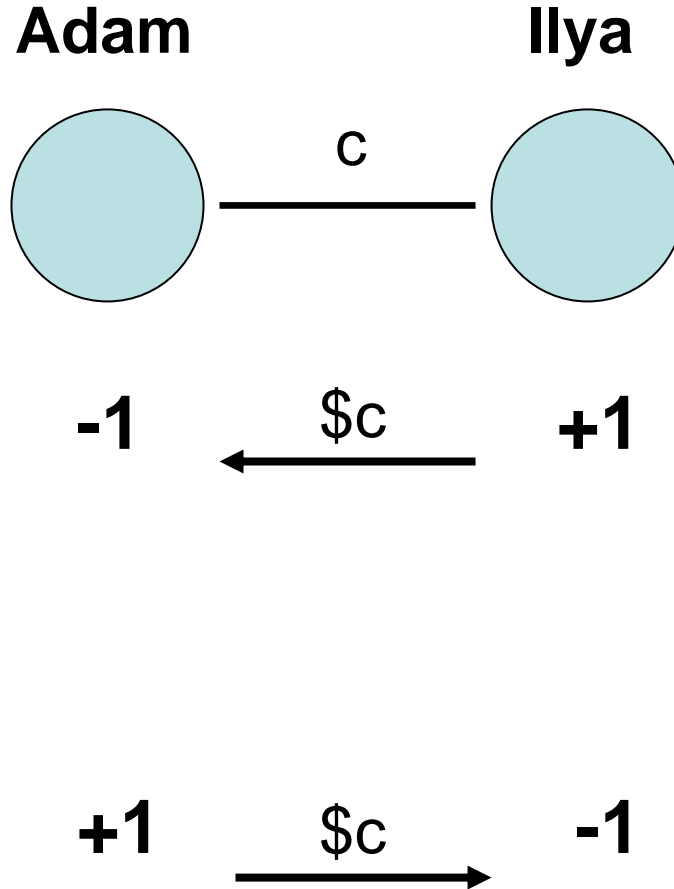
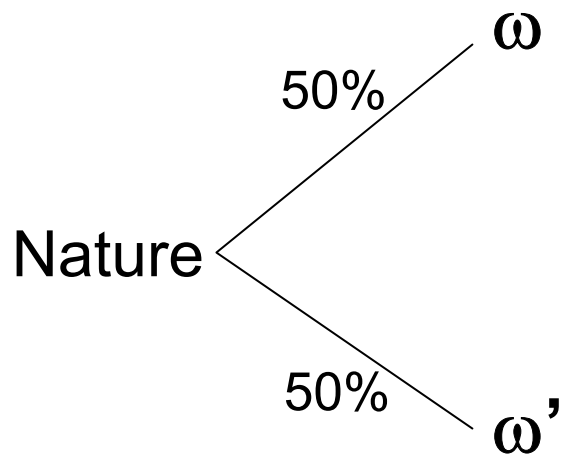
Suppose  $c < 1$ , then....



# Risksharing with informal enforcement

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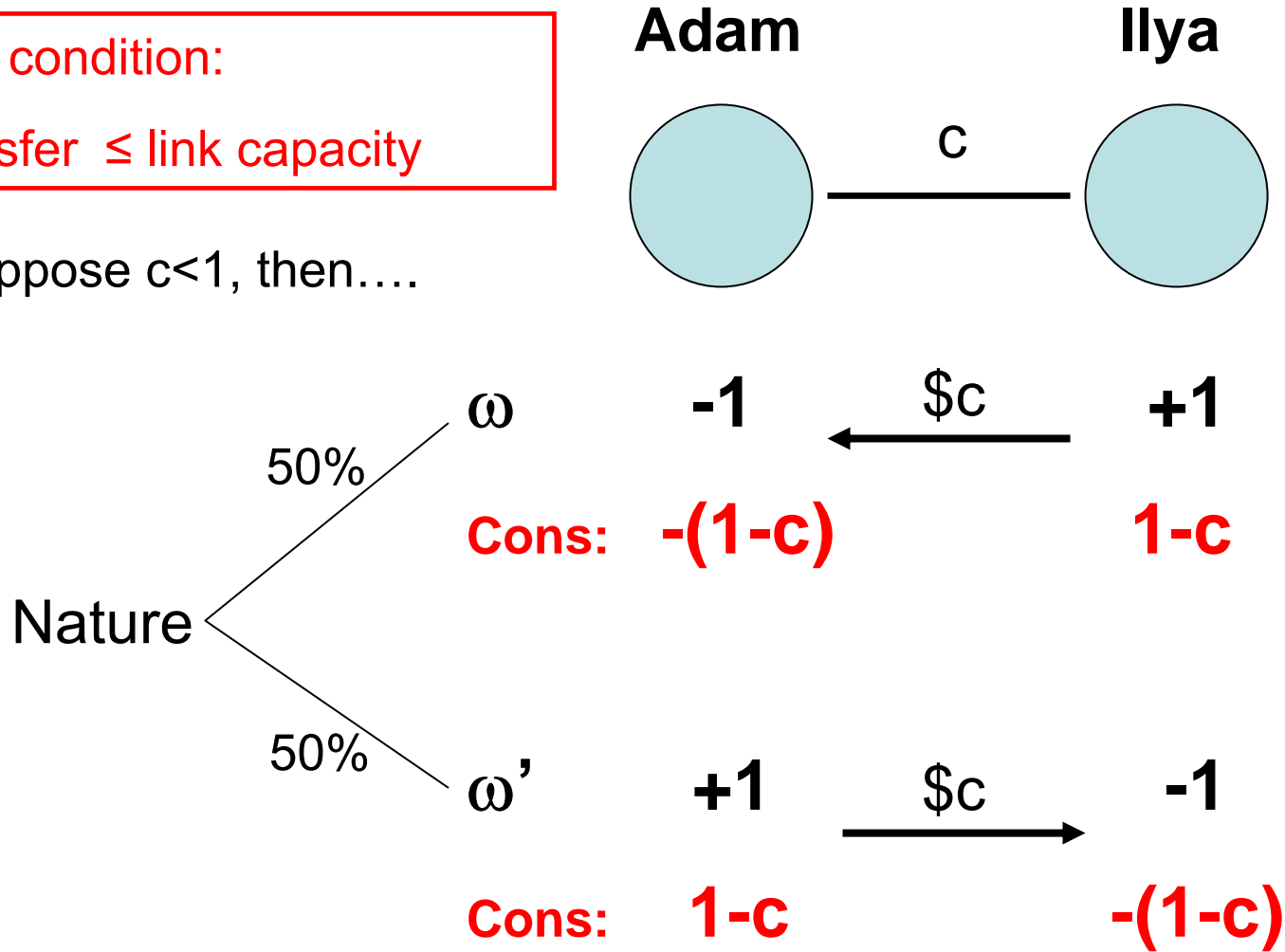
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# Risksharing with informal enforcement

Key condition:  
transfer  $\leq$  link capacity

Suppose  $c < 1$ , then....



# General model

- Consider  $N$  agents organized in exogenous social network  $G = (W, L)$ .
- A link represents friendship or business relationship between parties.
- Strength of relationships measured by non-negative capacity  $c : W \times W \rightarrow \mathbb{R}$ .
  - Assume that  $c(i, j) = c(j, i)$  and that  $c(i, j) = 0$  if  $(i, j) \notin L$ .

# Endowments and transfers

- A state of the world is characterized by a realization of agents' endowments:  $\{e_i\}_{i \in W}$ 
  - Endowments have a commonly known prior distribution.
  - All endowment realizations in product set of supports have positive probability.
- To share risk, agents ex ante agree on a set of transfers  $t_{ij}$  for all  $i, j \in W$  and for each state of the world.
  - Here  $t_{ij}$  is the net transfer to be paid by  $i$  to  $j$ .
  - By definition,  $t_{ij} = -t_{ji}$ .

# Contract enforcement

- After endowment realization, agents are expected to make promised transfers.
- If agent  $i$  fails to make promised transfer  $t_{ij}$ , he loses  $(i, j)$  link.
- These links disappear exogenously: friendly feelings no longer exist when somebody breaks a promise.
  - Can be microfounded: if  $j$  fails to receive  $t_{ij}$  payment, he concludes that  $j$  is no longer a friend, and stops interacting with him.

# Incentive compatible transfers

- Transfer arrangement is incentive compatible if agents make promised transfers for all realizations of uncertainty.
  - Better to pay transfer than to lose friend.

- **Proposition:** Arrangement is incentive compatible iff capacity constraint holds:

$$t_{ij} \leq c(i, j).$$

- Promised transfers should never exceed friendship value.

## 2. Model Analysis: Limits to risksharing

- **Key point:** one network statistic seems to govern limits to risksharing in many environments.

- For any subset of agents  $F$ , define *perimeter-to-area ratio* of  $F$  as

$$a[F] = \frac{c[F]}{|F|}$$

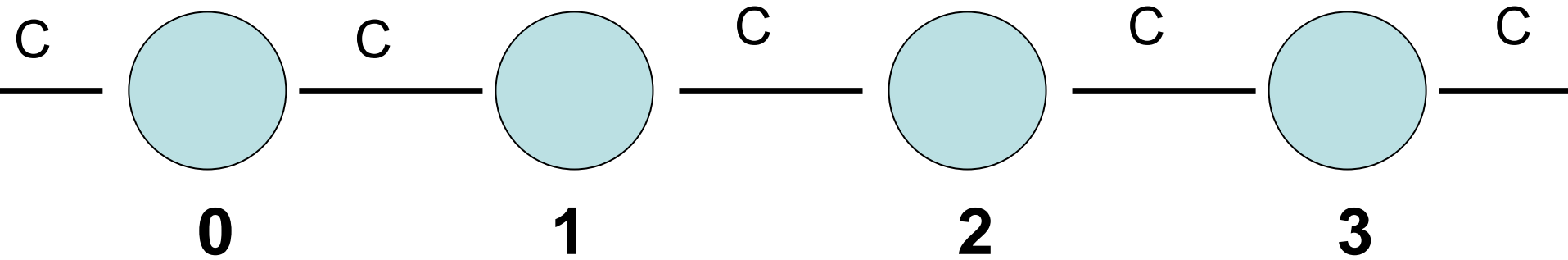
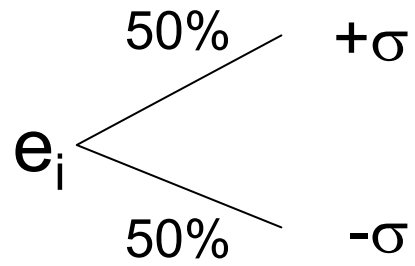
where  $c[F] = \sum_{i \in F, j \notin F} c(i, j)$  is the total capacity leaving  $F$ .

- Captures ability of set in offloading shocks relative to number of agents in set.
- For cubes of side  $n$  in the  $k$ -dimensional grid,  $a[F] = O(n^{-1})$ .

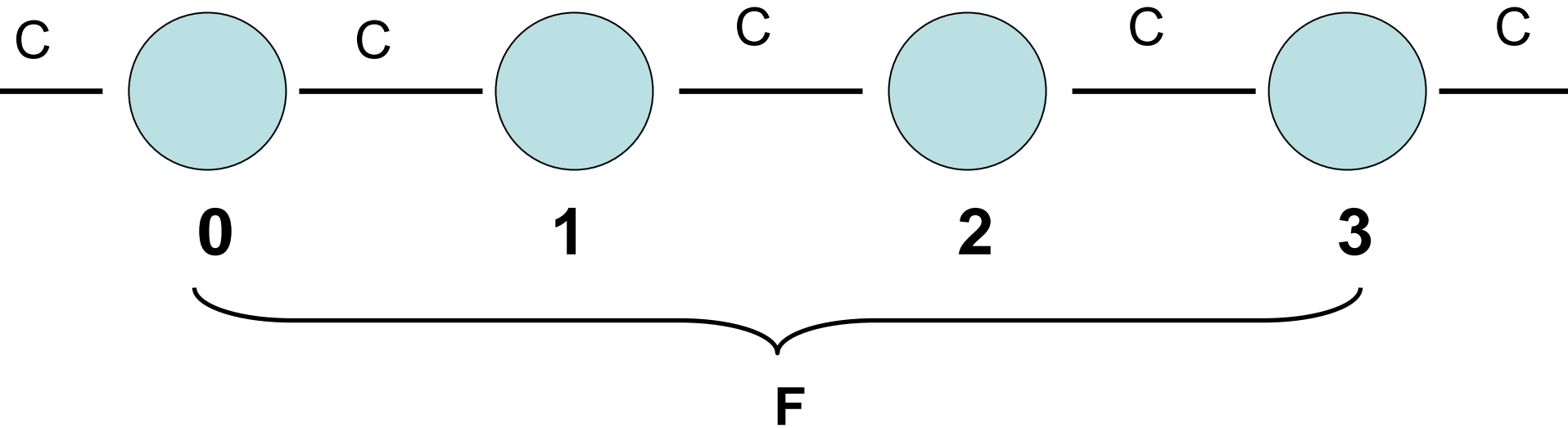
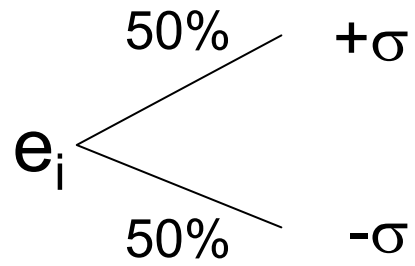
## Limits to risksharing

- Let  $\sigma_i$  be the standard deviation of  $e_i$ , and let  $\sigma = \min [\sigma_i]$ .
- **Proposition.** If  $a[F] < \sigma$  for some  $F$  with  $|F| \leq N/2$ , then no IC insurance arrangement is unconstrained Pareto-optimal, and there is an agent in  $F$  who is not fully insured.
  - If perimeter/area is not big enough, first best insurance cannot be obtained.
- Intuition: shocks accumulate over agents in  $F$ ; if perimeter is small, some accumulated shocks cannot fully leave  $F$ .

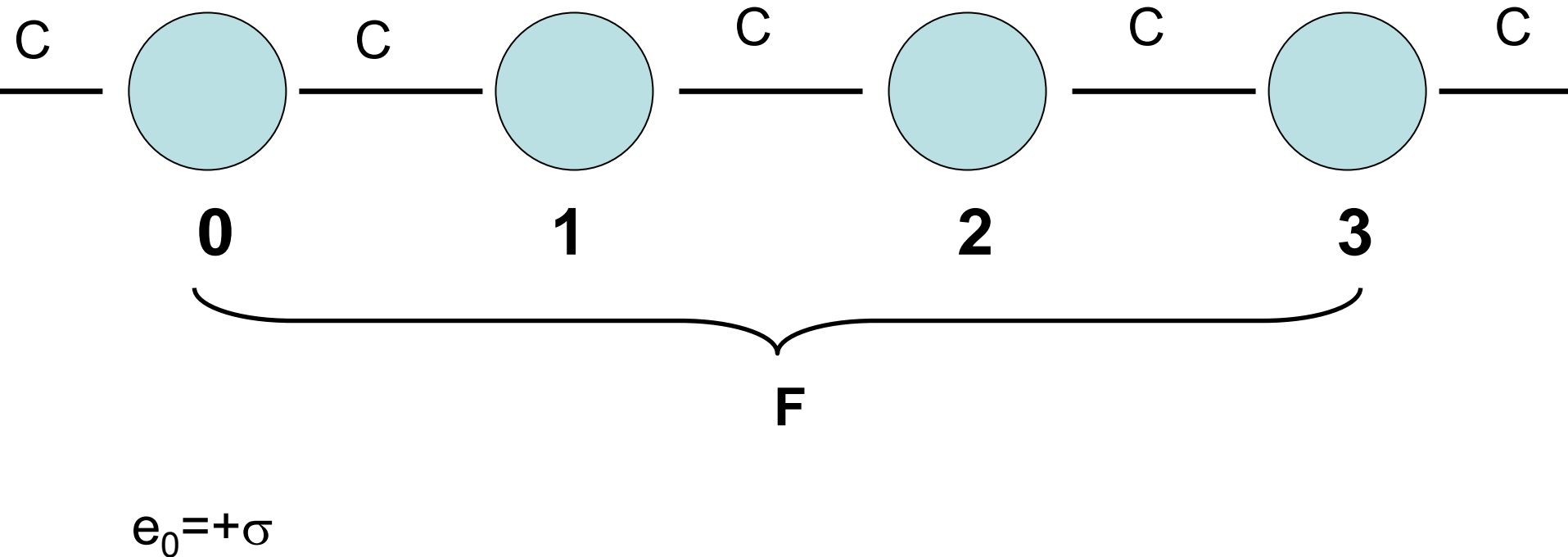
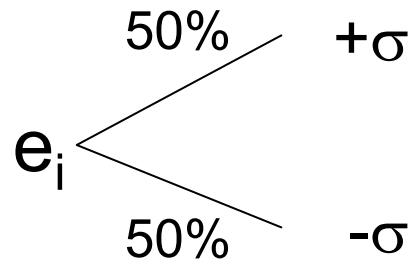
# Infinite line network



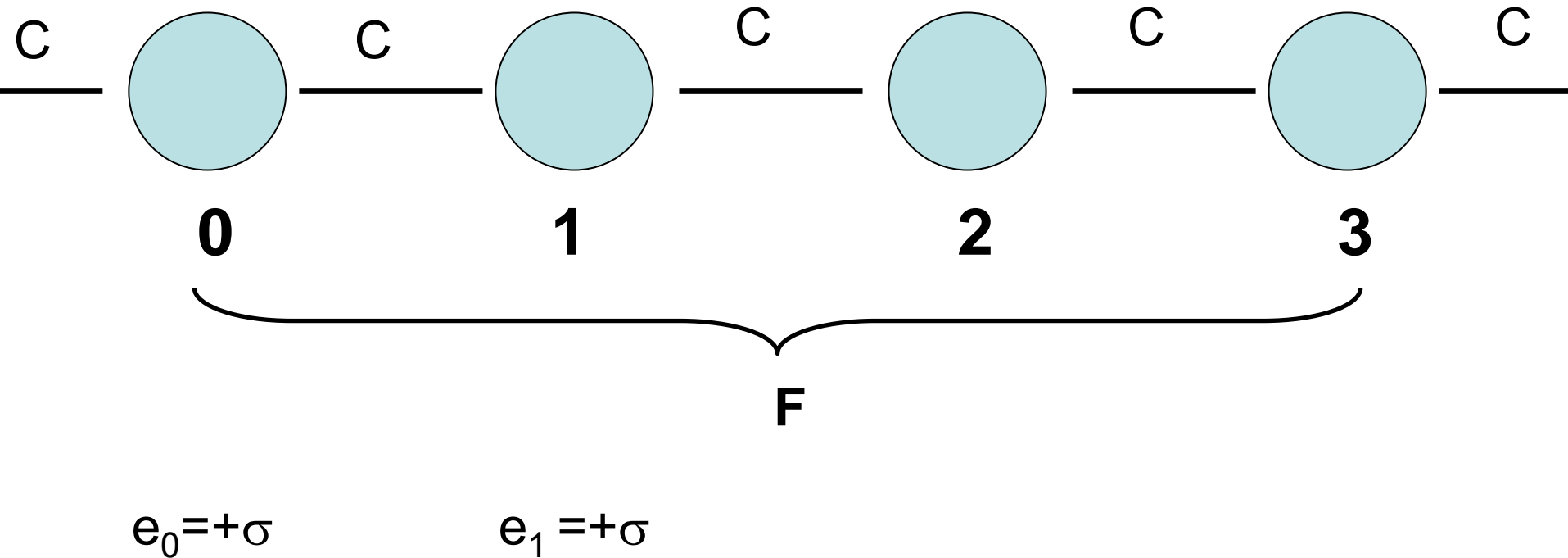
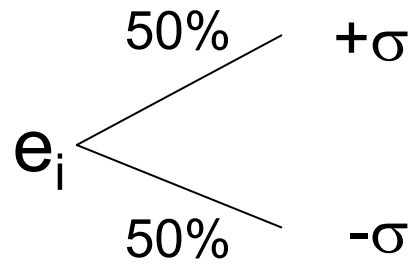
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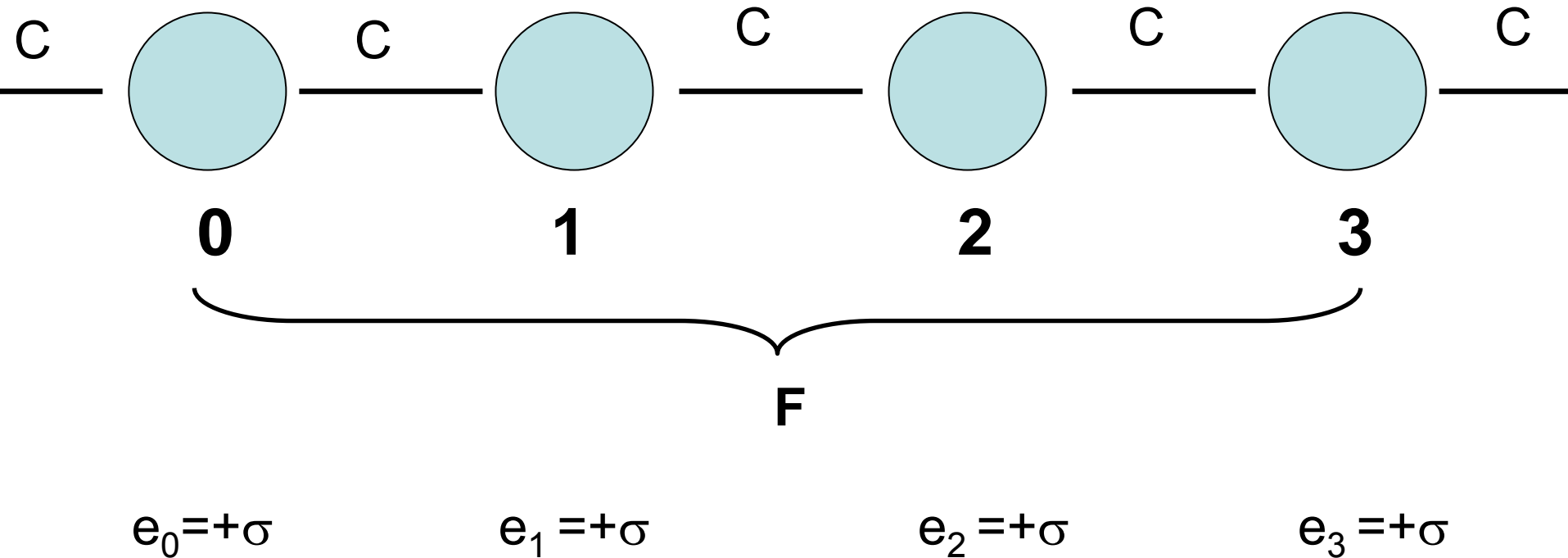
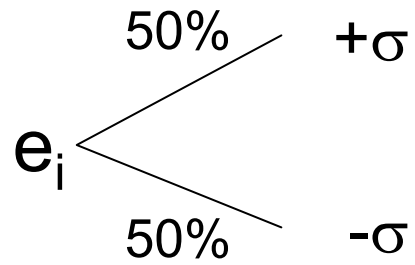
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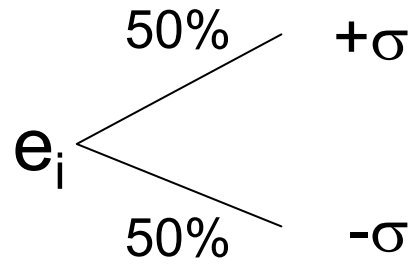
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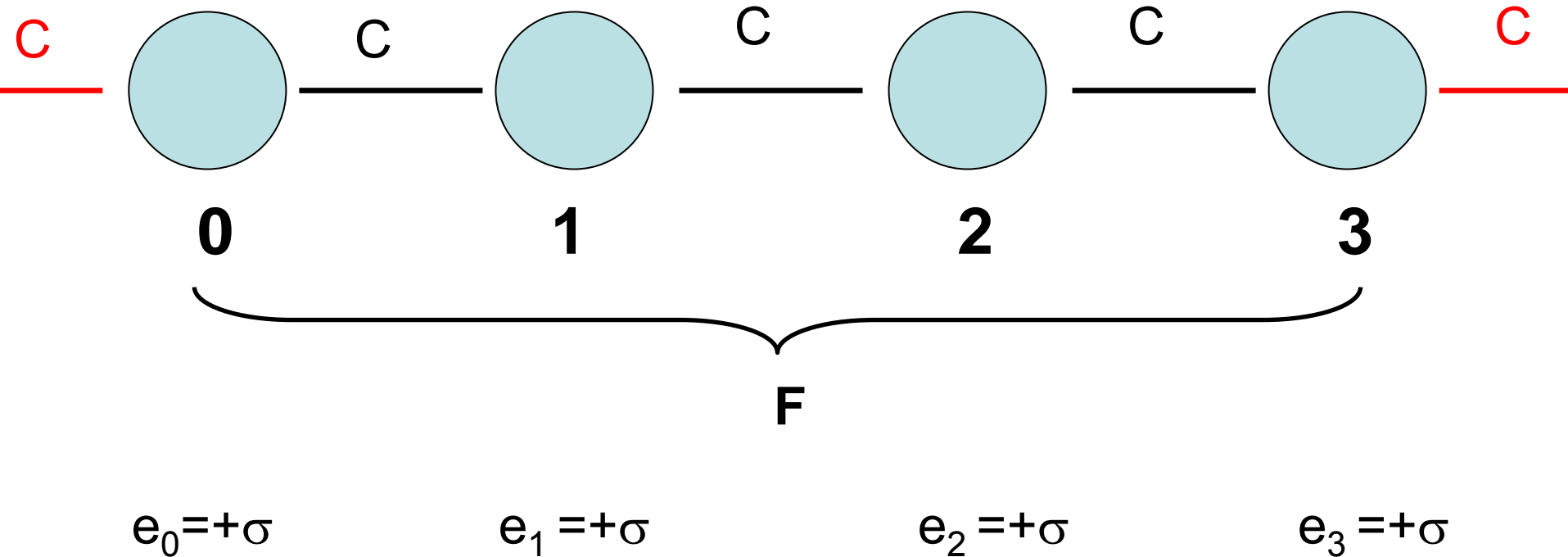
# Infinite line network



# Infinite line network



$$|F| \cdot \sigma \leq c[F]$$



# Imperfect risksharing: a measure of dispersion

- When Pareto-optimality fails, what degree of risksharing can be achieved?
- First need to develop some measure of the degree of risksharing.
- Define *cross-sectional dispersion of consumption* as square root of expected cross-sectional variance:

$$DISP = \mathbf{E} \left[ \frac{1}{N} \sum (x_i - \bar{x})^2 \right]^{1/2} .$$

- With i.i.d. shocks and no sharing (autarky),  $DISP = \sigma$ ;
- In equal sharing  $DISP = 0$ .

# Imperfect risksharing: a characterization result

- Let  $e_i$  be i.d.d. coin flips assuming values  $\sigma$  and  $-\sigma$  with equal probabilities, and assume

$$(1/N) \sum e_i = 0.$$

– Then  $DISP = \sigma$  in autarky.

- **Proposition** [Perimeter/area ratio and degree of risksharing]
  - (i) If  $a[F] \geq k$  for all  $F$  with  $|F| \leq N/2$ , then there exists an IC allocation where  $DISP = \sigma - k$ .
  - (ii) There exist networks and endowment distributions where this is sharp:  $DISP \geq \sigma - k$  for all IC arrangements.

# Imperfect risksharing: proof outline

- Fix network where  $a[F] \geq k$  for all  $F$  with  $|F| \leq N/2$ .
- In any endowment realization  $(e_i)$  there are  $N/2$  people with  $+\sigma$  and  $N/2$  people with  $-\sigma$ .
  - Need to transfer  $\$k$  from  $+\sigma$  guys to  $-\sigma$  guys.
- This is a flow problem  $\implies$  feasible if min cut  $\geq$  desired flow.
  - Perimeter/area inequality bounds are exactly what is needed to ensure this.

## Limits to risksharing: correlated shocks

- Suppose that agents are partitioned into disjoint sets (villages)  $F_m$ ,  $m = 1, 2, \dots, M$ .

- Agents experience both village-specific and idiosyncratic shocks:

$$e_i = s_m + \varepsilon_i$$

where  $s_m$  is common shock for village  $m$  and  $\varepsilon$  is i.i.d.

- To measure risksharing across villages, regress village per capita consumption on village-specific shock:

$$y_m = \alpha_m + \beta_m \cdot s_m + \nu_m.$$

- In full risksharing,  $\beta_m = O(1/N) \approx 0$ ; with no risksharing,  $\beta_m = 1$ .

## Correlated shocks continued

- High risksharing across villages means  $\beta$  close to zero; low risksharing means  $\beta$  close to 1.

- **Proposition.** We have

$$1 - \frac{a [F_m]}{\sigma_{sm}} \leq \beta_m \leq 1 + \frac{a [F_m]}{\sigma_{sm}}.$$

- When the perimeter/area ratio  $a [F_m]$  of a village is small,  $\beta$  is close to one.
- Village-specific shock cannot leave  $F_m$  through thin perimeter.

## Limits to risksharing: summary

- Perimeter-to-area ratio relative to  $\sigma$  governs limits to risksharing:
  1. When ratio small, full risksharing cannot be achieved;
  2. Ratio governs magnitude of cross-sectional cons dispersion;
  3. Ratio limits degree to which village-specific shocks can be shared.
- Consistent with evidence on limited risksharing:
  - Cochrane (1991): Large shocks (long unemployment spell, illness) are not fully insured.
  - Townsend (1994): More insurance within than across villages.

## 2B Constrained efficient allocations

- Can we learn more by focusing on “second best” arrangements?
- A risksharing arrangement is constrained efficient if it is Pareto-optimal subject to the IC constraints.
- Our approach:
  1. Second best arrangements are equivalent to solution of social planner’s problem with a set of weights.
  2. Use FOC of planner’s problem to characterize second-best allocations.

## FOC of planner's problem

- Assume all planner weights are equal.
- FOC with no IC constraints would be:  $U'_i = U'_j$  for all  $i, j$ .
- FOC with IC constraints: if  $U'_i < U'_j$  then  $t_{ij} = c(i, j)$ .
- Intuition:
  1. If marginal utility is not equalized, planner's objective can be improved by transferring a small amount from  $i$  to  $j$ .
  2. This is not possible  $\implies$  IC constraint must hold with equality.

# Endogenous risksharing “islands”

- Fix endowment realization  $\{e_i\}_{i \in W}$ .
- FOC can be used to partition network into connected components, such that
  1. Within a component, marginal utility is equalized;
  2. Across components, marginal utility differs and connecting links operate at full capacity.
- Shock are fully shared within an “island” but imperfectly shared across islands.

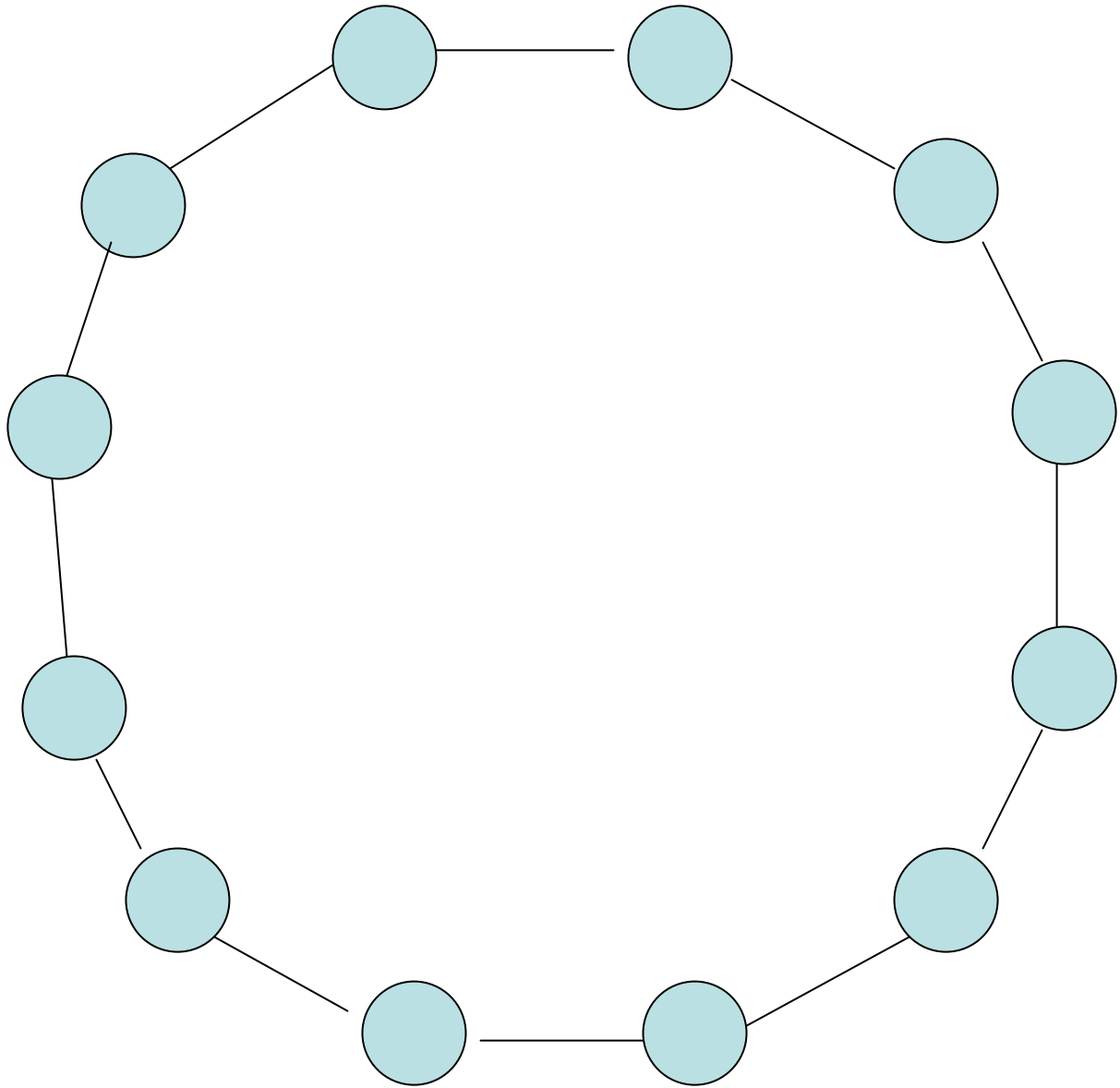
## Example: Circle network

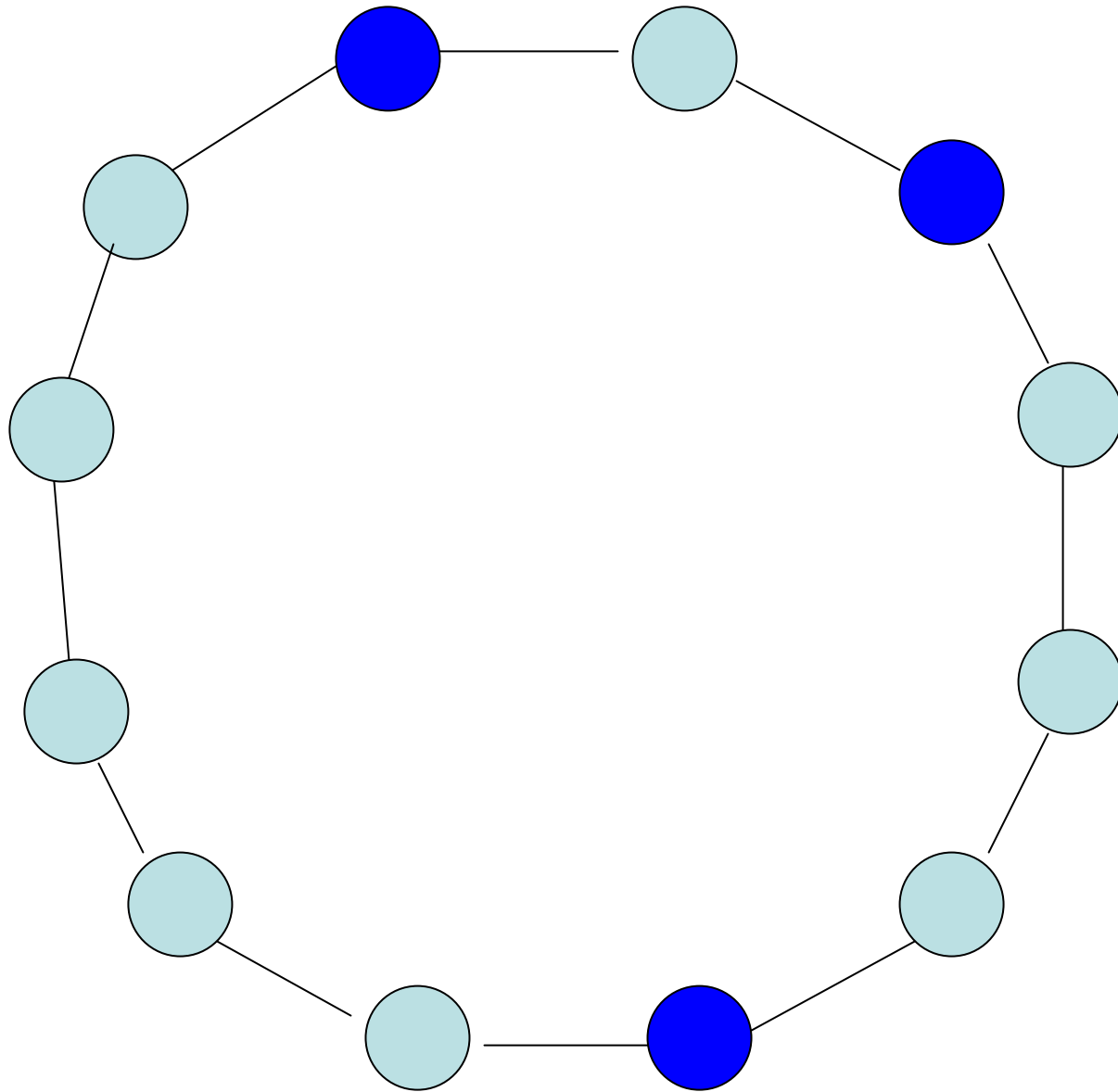
- Consider  $N$  agents organized in a circle w/ all link capacities =  $c$ .
- Suppose that  $e_i$  are identically distributed:

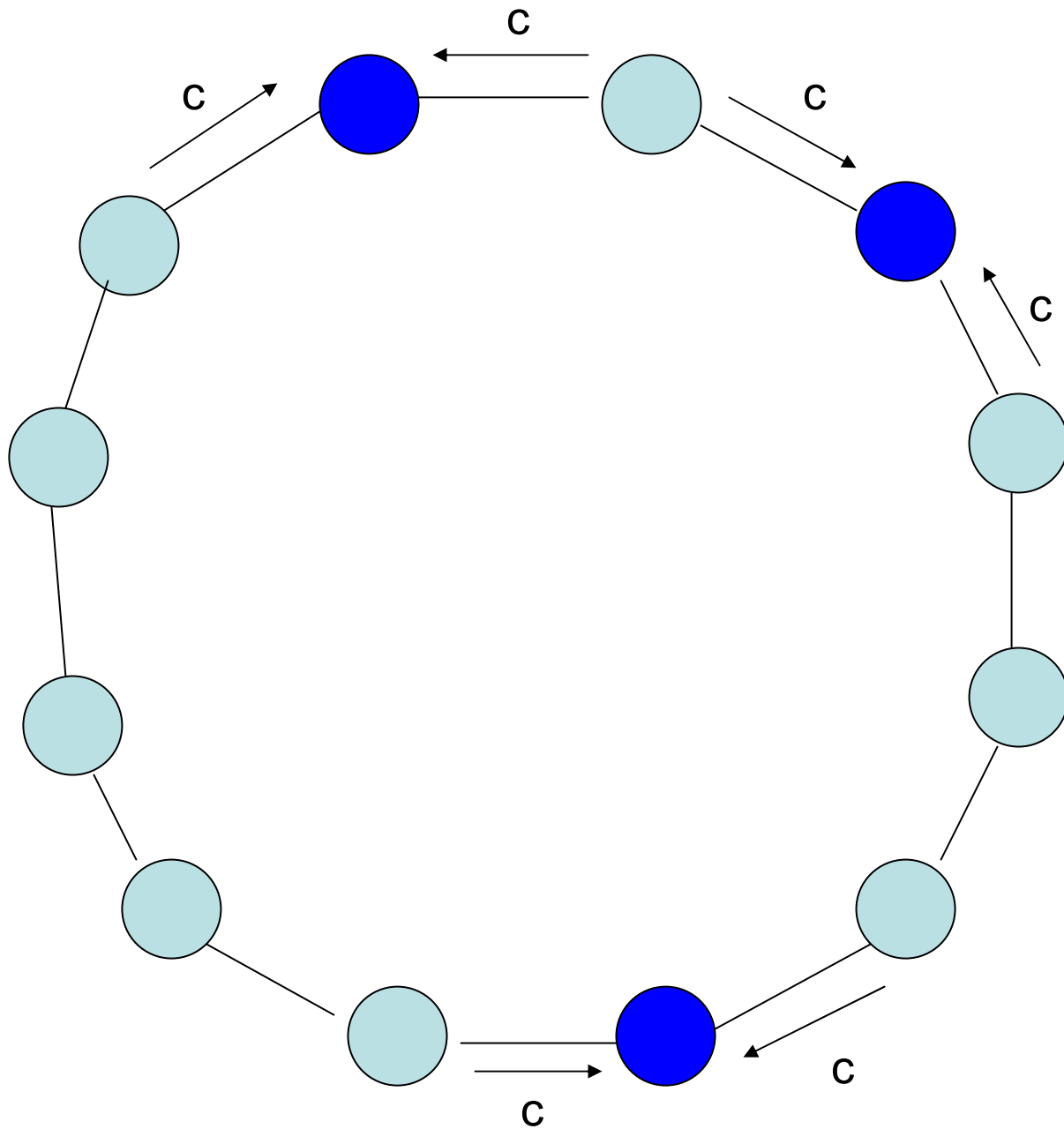
$$e_i = \begin{cases} -K & \text{with prob } p \\ 0 & \text{with prob } 1 - p \end{cases}$$

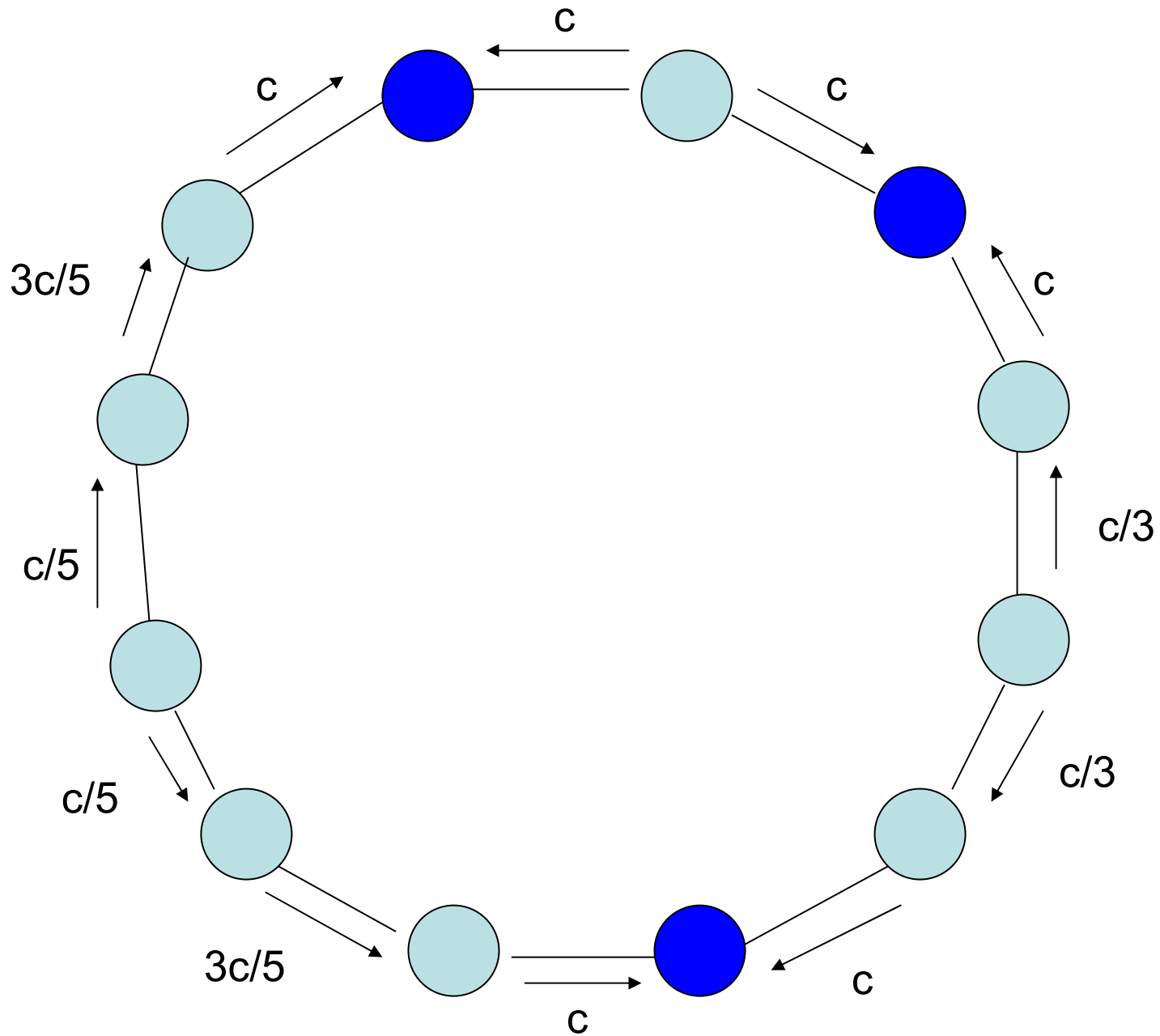
where  $K > 4c$ .

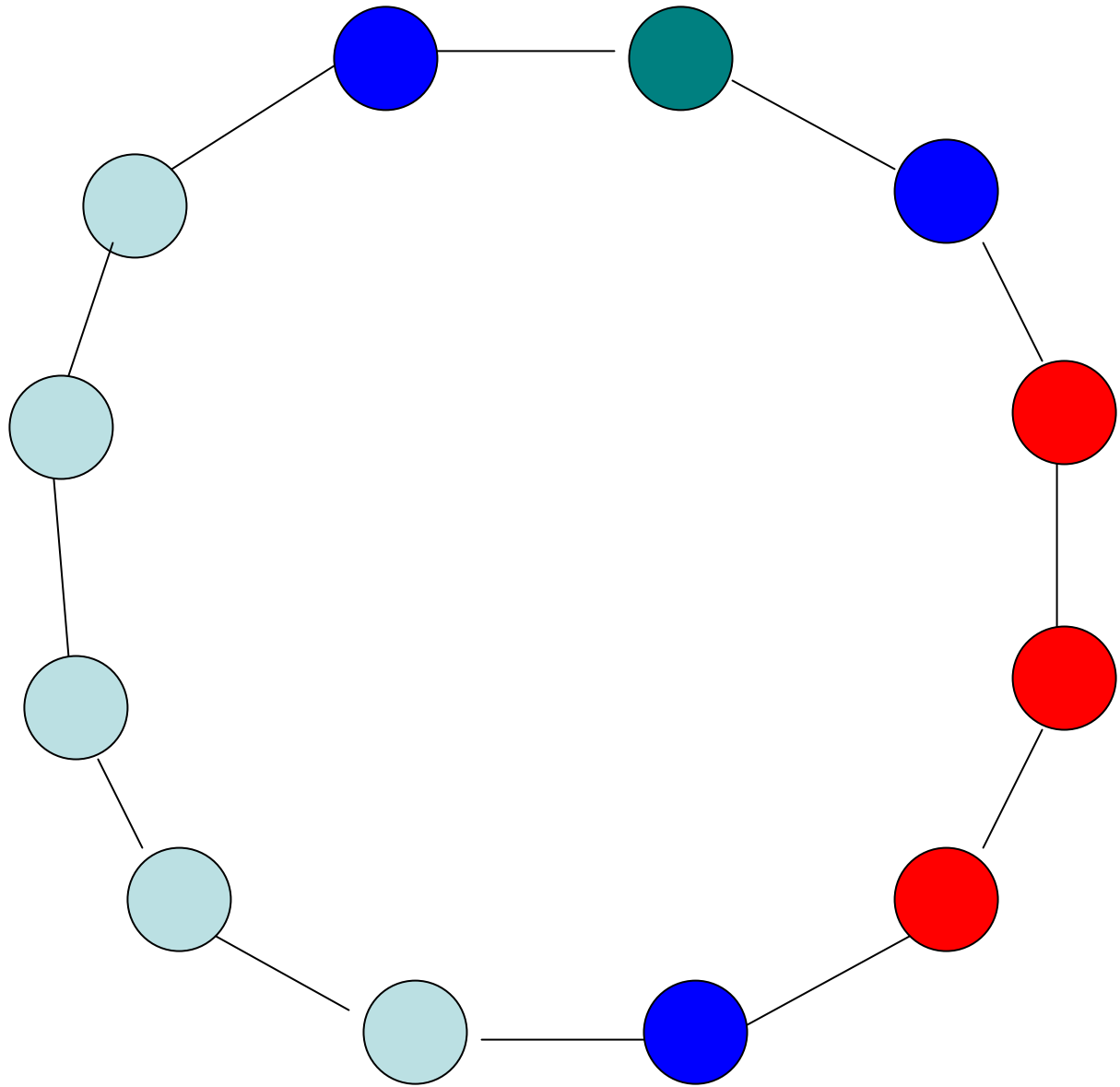
- $K$  represents adverse shocks such as unemployment or illness.
- Focus on efficient arrangement with equal planner weights.
  - If  $i$  and  $j$  are in the same equivalence class  $\implies x_i = x_j$ .









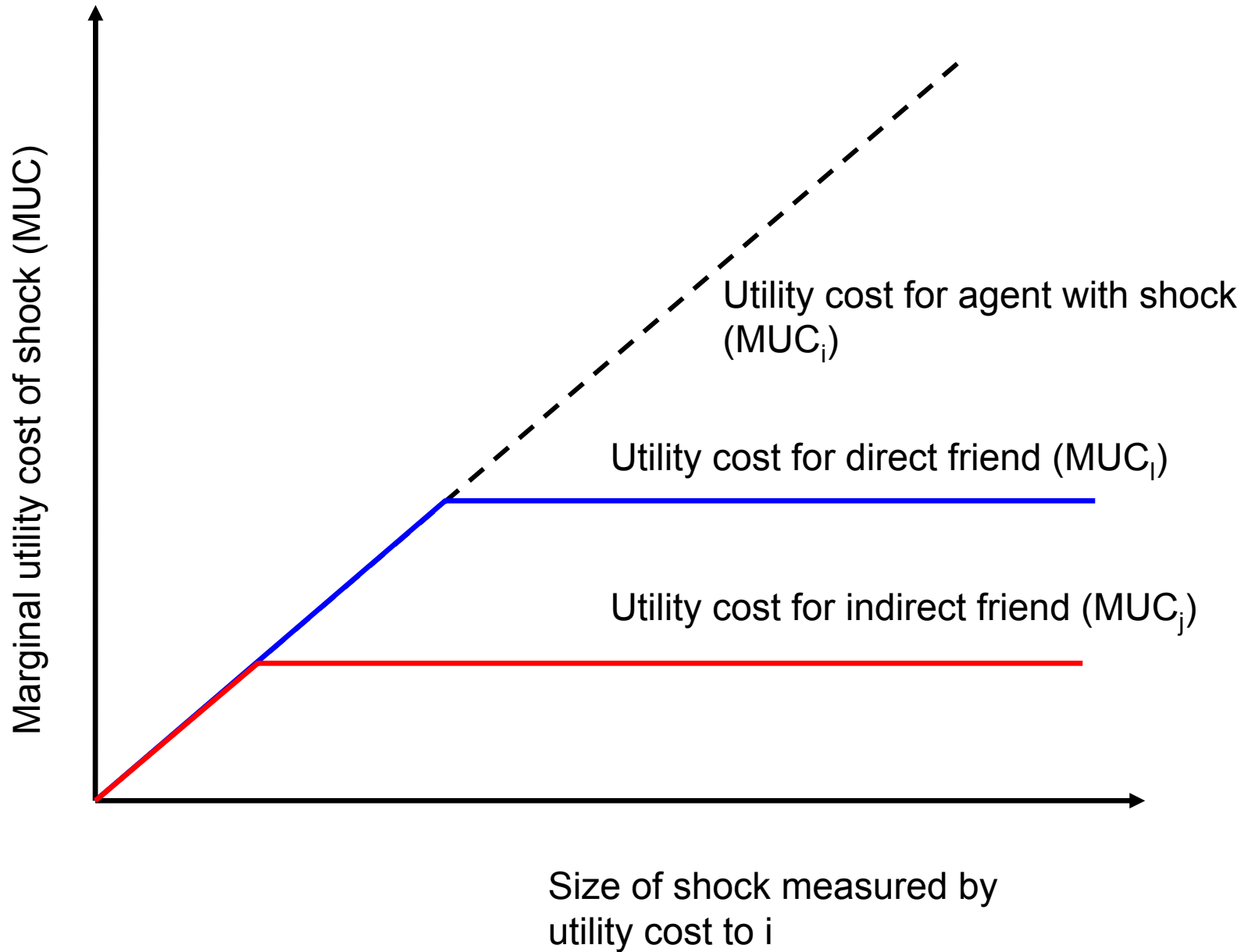


## Local sharing

- Island result suggests that socially closer agents share more risk with each other.
- To formalize this, fix realization  $e = (e_i)$  and define “bad shock”  $e'$  such that  $e'_i < e_i$  and  $e'_j = e_j$  for all  $j \neq i$ .
  - $(e')$  is an idiosyncratic negative shock to  $i$ .
- Measure impact of shock on agent  $j$  by marginal utility cost

$$MUC_j = \frac{U'_i(e')}{U'_i(e)}$$

where larger  $MUC_j$  corresponds to a more painful shock.



# Conclusion

- We developed a model of informal risksharing in social networks, where enforcement is provided by the “collateral value” of social links.
- Perimeter/area ratio governs limits to risksharing in many environments.
- In second best arrangements, agents organize in endogenous groups to share endowment shocks.
  - Socially close agents share more risk with each other.