Short Tutorial

Engineering Statistics for Multi-Object Tracking

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Top-Down vs. Bottom-Up Data Fusion

usual “bottom-up” approach to data fusion

- heuristically integrate a patchwork of optimal or heuristic algorithms that address specific functions (mostly done by operators)

“top-down,” system-level approach

- multiple sensors, multiple targets = single system

- specify states of system = multisensor-multitarget

- Bayesian-probabilistic formulation of problem requires random set theory

- reformulate in “Statistics 101, “engineering-friendly” terms

- low-level functions subsumed in optimally integrated algorithms

need tools for computational tractability!
Our Topic: “Finite-Set Statistics” (FISST)

• Systematic probabilistic foundation for multisensor, multitarget problems
• Preserves the “Statistics 101” formalism that engineers prefer
• Corrects unexpected difficulties in multitarget information fusion
• Potentially powerful new computational techniques (1st-order moment filtering)
• Unifies much of multisource, multitarget, multiplatform data fusion

unified Bayesian data fusion
# FISST-Related Books and Monographs

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<td><strong>MATHEMATICS OF DATA FUSION</strong></td>
<td><strong>Random Sets</strong></td>
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<td><strong>HANDBOOK OF MULTISENSOR DATA FUSION</strong></td>
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## Scientific Workshops

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Topics

1. Introducing “Dr. Dogbert,” Critic of FISST
2. What is “engineering statistics”? 
3. Finite-set statistics (FISST)
4. Multitarget Bayes Filtering
5. Brief introduction to applications
The multisource-multitarget engineering problems addressed by FISST actually require nothing more complicated than Bayes’ rule…

…which means that FISST is of mere theoretical interest at best. At worst, it is nothing more than pointless mathematical obfuscation.

multitarget calculus
multitarget statistics
multitarget modeling

What is extraordinary about this assertion is not the ignorance it displays regarding FISST but, rather, the ignorance it displays regarding Bayes’Rule!
The optimality and simplicity of the Bayesian framework can be taken for granted only within the confines of standard applications addressed by standard textbooks. When one ventures out of these confines one must exercise proper engineering prudence—which includes verifying that standard textbook assumptions still apply.

This is especially true in multitarget problems!
How Multi- and Single-Object Statistics Differ

Bayes’ rule

multitarget maximum likelihood estimator (MLE) is defined but multitarget maximum a posteriori (MAP) estimator is not!

new multitarget state estimators must be defined and proved optimal!

multitarget Shannon entropy cannot be defined!

multitarget L² (i.e., RMS distance) metric cannot be defined!

need explicit methods for constructing provably true multisensor-multitarget likelihood functions from explicit sensor models!

need explicit methods for constructing provably true multitarget Markov transition densities from explicit motion models!

multitarget expected values are not defined!
I, the great Dr. Dogbert, have discovered a powerful new research strategy! Strip off the mathematics that makes FISST systematic, general, and rigorous, copy its main ideas, change notation, and rename it Dogbert Theory!

This is a great advance because it’s much simpler: It doesn’t need all that ugly math (Blech!) that no one understands anyway!

A mere change of name and notation does not constitute a great advance.

Dogbert Theory is straightforward precisely because it avoids specifying explicit general methods and foundations—and yet is still fully general and derived from first principles!

It is easy to claim invention of a simple and yet elastically all-subsuming theory if one does so by neglecting, avoiding, or glossing over the specifics required to actually substantiate puffed-up boasts of generality and “first principles”.
What is “Engineering Statistics”?

- Engineering statistics, like all engineering mathematics, is a tool and not an end in itself. It must have two (inherently conflicting) characteristics:
  - **Trustworthiness**: Constructed upon systematic, reliable mathematical foundations, to which we can appeal when the going gets rough.
  - **Fire and forget**: These foundations can be safely neglected in most applications, leaving a parsimonious and serviceable mathematical machinery in their place.

- The “Bar-Shalom Test” defines the dividing line between the serviceable and the simplistic:
  - “Things should be as simple as possible—but no simpler!”
    - Y. Bar-Shalom, quoting Einstein

- If foundations are:
  - so mathematically complex that they cannot be taken for granted in most situations, then they are shackles and not foundations!
  - so simple that they repeatedly lead us into engineering blunders, then they are simplistic, not simple!
Progress in single-sensor, single-target applications has been greatly facilitated by the existence of a systematic, rigorous, and yet practical engineering statistics that supports the development of new concepts in this field.

\[
Z_k = h_k(x) + W_k
\]

\[
X_{k+1} = \Phi_k(x) + V_k
\]

- sensor measurement model
- target Markov motion model

\[
p_k(S) = \Pr(Z_k \in S)
\]

\[
p_{k+1|k}(S) = \Pr(X_{k+1} \in S)
\]

- probability mass function
- probability mass function

\[
f_k(z|x)
\]

\[
f_{k+1|k}(y|x)
\]

- likelihood function
- Markov transition density

\[
E[Z] \quad E[X]
\]

- expectations & other moments
- Bayes-optimal state estimators

\[
\frac{d}{dx}
\]

- integro-differential calculus

\[
||z_1 - z_2||
\]

\[
||x_1 - x_2||
\]

- miss distance
What is Single-Target Bayes Statistics?

Classical statistics: unknown state-vector $x$ of the target is a fixed parameter.

Bayesian statistics: unknown state is always a random vector $X$ even if we do not explicitly know either it or its distribution $f_0(X)$ (the prior). The single-sensor, single-target likelihood function is a cond. prob. density $L_z(x) = f(z|x) = \Pr(Z=z|X=x)$.

*equalities temporarily assume discrete state & observations spaces for ease of argument*
specify single-sensor/target measurement space and its measure structure

\[ f(z|x) = \Pr(Z=z|X=x) \]

specify single-target state space and its measure structure

\[ f_0(x) = \Pr(X=x) \]

random variables (e.g. random vectors)

signifies a random draw from a single measurement space

signifies a random draw from a single state-space
The Recursive Bayes Filter

1. *time-update step via prediction integral*

\[ f_{k+1|k}(y|Z^k) = \int f_{k+1|k}(y|x) f_{k|k}(x|Z^k) dx \]

**prediction of posterior to time of new datum**  
**current posterior, conditioned on old data-stream**

\[ Z^k = \{ z_1, \ldots, z_k \} \]

**true Markov transition density**

\[ X_{k+1|k} = \Phi_k(x) + V_k \quad (motion \ model) \]

\[ f_{k+1|k}(y|x) = f_{V_k}(y - \Phi_k(x)) \]
The Recursive Bayes Filter

2. data-update step via Bayes’ rule

new posterior, conditioned on all data \( Z^{k+1} = \{z_1, \ldots, z_{k+1} \} \)

prior = predicted posterior

\[
f_{k+1|k+1}(x|Z^{k+1}) \propto f_{k+1}(z_{k+1}|x) f_{k+1|k}(x|Z^k)
\]

assuming \( f_{k+1}(z|x, Z^k) = f_{k+1}(z|x) \)

true likelihood function

\[
Z_k = h_k(x) + W_k \quad \text{(measurement model)}
\]

\[
f_k(z|x) = f_{W_k}(z - h_k(x))
\]
The Recursive Bayes Filter

3. estimation step via Bayes-optimal state estimator

\[ \hat{x}_{k+1|k+1} = \left\{ \begin{array}{l} \arg\sup_x f_{k+1|k+1}(x|Z^{k+1}) \\ \text{maximum a posteriori (MAP)} \end{array} \right. \]

\[ \int x f_{k+1|k+1}(x|Z^{k+1}) dx \]

expected a posteriori (EAP)

\[ X_{k+1|k+1} \]

current best estimate of \( x_{k+1} \)
Basic Issues in Single-Target Bayes Filtering

- **without “true” likelihood for both target & background, Bayes-optimal claim is hollow**
- **“over-tuned” likelihoods may be non-robust w/r/t deviations between model & reality**
- **poor motion-model selection leads to performance degradation**
- **without “true” Markov density, good model selection is wasted**

**Sensor models**

\[ f(z|x) \]

**Motion models**

\[ f_{k+1|k}(x|y) \]

**State estimation/convergence**

\[ \hat{x}_{k|k} \]

**Computability**

\[ f_k(y|Z_k) \]

**Filtering equations are computationally nasty**

- **poorly chosen state estimators may have hidden inefficiencies (erratic, inaccurate, slowly-convergent, etc.)**
- **without guaranteed convergence, approx. filter may be unstable, non-robust**
- **“textbook” approaches create computational inefficiencies: e.g., numerical instability due to central finite-difference schemes**
Bayes-Optimal State Estimators

pre-specified cost function

\[ C(x, y) \]

\[ \hat{x}(z_1, \ldots, z_m) \]

state estimator: family of state-valued functions of the observations \( z_1, \ldots, z_m, m > 0 \)

Bayes risk = expected value of the total posterior cost

\[ R_C(\hat{x}, m) \propto \int C(x, \hat{x}(z_1, \ldots, z_m)) f(z_1, \ldots, z_m | x) f_0(x) dz_1 \cdots dz_m dx \]

state estimator \( \hat{x} \) is “Bayes-optimal” with respect to cost \( C \) if it minimizes the Bayes risk

without a Bayes-optimal estimator, we aren’t Bayes-optimal!
**OBSERVATIONS**

- **measurement model**
  - random observation
  - deterministic measurement
  - sensor noise process
  
  \[ Z_k = h_k(x) + W_k \]

- **probability-mass function**
  
  \[ p_k(S|x) = \Pr(Z_k \in S) \]
  
  = probability that observation is in \( S \)

- **true likelihood function**
  
  \[ f_k(z|x) = \frac{\delta p_k}{\delta z} \]
  
  = Radon-Nikodým derivative of \( p_k \)

---

**Bayesian modeling**

- **Step 1:** construct model

**MOTION**

- **motion model**
  - random state of moved target
  - deterministic motion model
  - motion noise process
  
  \[ X_{k+1|k} = \Phi_{k+1|k}(x) + V_k \]

- **probability-mass function**
  
  \[ p_{k+1|k}(S|x) = \Pr(X_{k+1|k} \in S) \]
  
  = probability that moved target is in \( S \)

- **true Markov transition density**
  
  \[ f_{k+1|k}(y|x) = \frac{\delta p_{k+1|k}}{\delta y} \]
  
  = Radon-Nikodým derivative of \( p_{k+1|k} \)
Finite-Set Statistics (FISST)

- One would expect that *multisensor, multitarget* applications rest upon a similar engineering statistics

- *Not so*, despite long existence of *point process theory* (PPT)

- FISST is in part an “engineering friendly” version of PPT

multisensor-multitarget observations

random observation-set

multitarget states

random state-set

multitarget measurement model

multitarget Markov motion model

belief-mass function

belief-mass function

FISST integro-differential calculus

Bayes-optimal filtering

multitarget posterior & prior densities

multitarget Markov transition density

**formal Bayes modeling?**
FISST, I

systematic, probabilistic framework for modeling uncertainties in data

likelihood function, $f(z|x)$

$\int f(z|x)dz = 1$

common probabilistic framework: random closed sets, $\Theta$

generalized likelihood function, $\rho(z|x)$

$\int \rho(z|x)dz = \infty$

random set model

random set model

random set model

statistical

geradar report

fuzzy

imprecise

contingent

English-language report
data link attribute

rule

poorly characterized likelihood
Dealing With Ill-Characterized Data

Observation = “Gustav is NEAR the tower”

interpretations of “NEAR”

(random set model $\Theta$ of natural language statement)

probabilities for each interpretation
FISST, II

reformulate multi-object problem as generalized single-object problem

\[ \sum_{\text{random observation-set}} \Xi \]

\[ \left\{ X_1, \ldots, X_n \right\} \]

\[ \left\{ X^*_1, \ldots, X^*_s \right\} \]

\[ Z = \left\{ z_1, \ldots, z_m \right\} \]
**FISST, III**

*multisource-multitarget “Statistics 101”*

---

**single-sensor/target**

- sensor
- target
- vector observation, $z$
- vector state, $x$

- derivative, $dp_z/dz$
- integral, $\int f(x) \, dx$

- prob.-mass func., $p_z(S)$
- likelihood, $f_z(z|x)$
- prior PDF, $f_0(x)$

- information theory
- filtering theory

---

**multi-sensor/target**

- meta-sensor
- meta-target
- finite-set observation, $Z$
- finite-set state, $X$

- set derivative, $\delta \beta_\Sigma / \delta Z$
- set integral, $\int f(Z) \, \delta Z$

- belief-mass func., $\beta_\Sigma(S)$
- multitarget likelihood, $f_\Sigma(Z|X)$
- multitarget prior PDF, $f_0(X)$

- multitarget information theory
- multitarget filtering theory

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*Almost-parallel Worlds Principle (APWOP)*
FISST, IV

systematic, rigorous foundation for:
- ill-characterized data & likelihoods
- multitarget filtering and estimation
- multigroup filtering & sensor management
- computational approximation

unified Bayesian data fusion
Classical statistics: unknown state-set $X$ of the target is a fixed parameter

Bayesian statistics: unknown state-set is always a random finite set $\Xi$ even if we do not explicitly know either it or its distribution $f_0(X)$ (the prior). The multisensor-multitarget likelihood function is a cond. prob. density

$\sum\Xi$ random state-set  
$\Xi$ random observation-set  
observation space, $\sum$  
state space, $X$  

multisensor-multitarget likelihood function* 
$L_{\sum}(X) = f(Z|X) = \Pr(\sum = Z | \Xi = X)$

$f_0(X) = \Pr(\Xi = X)$  
multitarget prior distribution* of $\Xi$

* equalities temporarily assume discrete state & observationspaces for ease of argument
Multitarget Bayes Statistics: Distributions

Multitarget likelihoods and distributions must have a specific form.

\[ f(Z|X) = \Pr(\Sigma = Z|\Xi = X) \]

\[ f_0(X) = \Pr(\Xi = X) \]

strictly speaking: re-writing \( f_0(X) \) as a family of ordinary densities (one for each choice of target number) is not Bayesian because then it does not describe a random draw from a single multitarget state-space:

\[ f_0^0(\emptyset), \ f_0^1(x_1), \ f_0^2(x_1, x_2), \ldots, \ f_0^n(x_1, \ldots, x_n), \ldots \]
Point Processes / Random Finite Sets

**a point process is a random finite multi-set**

- **random finite sets are best for engineering**
- **are geometric and preserve "Statistics 101" formalism**
- **other notations add complexity, no new information, and lose simple/useful set-theoretic tools**
Set integral:

\[ \int f(Y) \, \delta Y = f(\emptyset) + \sum_{n=1}^{\infty} \int f(\{y_1, \ldots, y_n\}) \, dy_1 \cdots dy_n \]

- Sum over all possible numbers of targets
- Sum over all multi-object states with \( n \) objects
Set Derivative (Multi-Object Derivative)

Frechét derivative of a functional $G[h]$ (of a function-variable $h(x)$):

$$\frac{\partial G}{\partial g}[h] = \lim_{\varepsilon \to 0} \frac{G[h + \varepsilon \cdot g] - G[h]}{\varepsilon}$$

$g \to \frac{\partial F}{\partial g}[h]$ is linear, cts.

Functional derivative (from physics)

$$\frac{\partial G}{\partial \delta_x}[h] = \lim_{\varepsilon \to 0} \frac{G[h + \varepsilon \cdot \delta_x] - G[h]}{\varepsilon}$$

Dirac delta function

Set derivative of a set-function $\beta(S) = G[1_S]$:

$$\frac{\delta \beta}{\delta X}(S) = \frac{\delta^n \beta}{\delta x_n \cdots \delta x_1}(S) = \frac{\partial G}{\partial \delta x_n \cdots \partial \delta x_1}[1_S]$$

$X = \{x_1, \ldots, x_n\}$
**Probability Law of a Random Finite Set**

<table>
<thead>
<tr>
<th>Probability generating functional (p.g.fl.)</th>
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<tr>
<td>$G_\Xi[h] = E[h^\Xi]$</td>
</tr>
<tr>
<td>$h^X = \prod_{x \in X} h(x)$</td>
</tr>
<tr>
<td>$h = \text{bounded real-valued test function}$</td>
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**Belief-mass function**

$$\beta_\Xi(S) = G_\Xi[1_S] = \Pr(\Xi \subseteq S)$$

**Choquet capacity theorem**

**Multitarget probability distribution**

$$f_\Xi(X) = \frac{\delta \beta_\Xi}{\delta X}(\emptyset) = \Pr(\Xi = X)$$

$$X = \{x_1, \ldots, x_n\}$$
Multitarget Posterior Density Functions

\[ \int f_{k|k}(X \mid Z^{(k)}) \delta X = 1 \]

(normality condition)

\[ f_{k|k}(X \mid Z^{(k)}) \]

multitarget posterior

multitarget state

\[ f_{k|k}(\emptyset \mid Z^{(k)}) \]
(no targets)

\[ f_{k|k}(\{x_1\} \mid Z^{(k)}) \]
(one target)

\[ f_{k|k}(\{x_1, x_2\} \mid Z^{(k)}) \]
(two targets)

... 

\[ f_{k|k}(\{x_1, \ldots, x_n\} \mid Z^{(k)}) \]
(n targets)

measurement-stream

\[ Z^{(k)} = \{Z_1, \ldots, Z_k\} \]

multisensor-multitarget measurements:

\[ Z_k = \{z_1, \ldots, z_{m(k)}\} \]

individual measurements collected at time \( k \)
random observation-sets $Z$ produced by targets

The Multitarget Bayes Filter

$\mathbf{Ξ}_{k|k}$

$\mathbf{Ξ}_{k+1|k+1}$

$\mathbf{Ξ}_{k+1|k}$

random state-set

state space

observation space

random state-set

multitarget Bayes filter

usually computationally intractable!
Issues in Multitarget Bayes Filtering

what does “true” multisensor-multitarget likelihood even mean?

how do we construct multisensor-multitarget measurement models and multisensor-multitarget likelihood functions?

multitarget motion models must account for changes and correlations in target number and motions

how do we construct multitarget motion models and true multitarget Markov densities?

what does “true” multitarget Markov density even mean?

FISST explicitly addresses these problems

must define new multitarget state estimators and show well-behaved

new mathematical tools even more essential than in single-target case

multitarget analogs of usual Bayes-optimal estimators are not even defined!

state estimation computability
**OBSERVATIONS**

Multitarget meas. model

- \( \Sigma_k = T_k(X) \cup C_k \)

Belief-mass function

- \( \beta_k(S|X) = \Pr(\Sigma_k \subseteq S) \)
  
  = probability that observation is in \( S \)

True likelihood function

- \( f_k(Z|X) = \frac{\delta \beta_k}{\delta Z}(\emptyset|X) \)
  
  = set derivative of \( \beta_k \)

**Bayesian modeling**

Step 1: construct model

- Step 2: use belief-mass function to describe model statistics

**MOTION**

Multitarget motion model

- \( \Xi_{k+1|k} = \Phi_{k+1|k}(X) \cup B_k \)

Belief-mass function

- \( \beta_{k+1|k}(S|X) = \Pr(\Xi_{k+1|k} \subseteq S) \)
  
  = probability that moved target is in \( S \)

True Markov transition density

- \( f_{k+1|k}(Y|X) = \frac{\delta \beta_{k+1|k}}{\delta Y}(\emptyset|X) \)
  
  = set derivative of \( \beta_{k+1|k} \)
### Failure of Classical Bayes Estimators

#### Simple Posterior Distribution

<table>
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<tr>
<th>State</th>
<th>Probability</th>
<th>Posterior Probability</th>
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<tbody>
<tr>
<td>∅ (no-target state)</td>
<td>1/2</td>
<td>( f(x) = \frac{1}{4} \text{ km}^{-1} )</td>
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#### Naïve MAP

- **Compare**
  - \( f(∅) = \frac{1}{2} \)
  - \( f(x) = \frac{1}{4} \text{ km}^{-1} \)
  - *incommensurable!*

#### Naïve EAP

\[
\int X f(X) \, \delta X = ∅ \cdot F(∅) + \int_{0}^{2} x f(x) \, dx = \frac{1}{2} \cdot (∅ + 1 \text{ km})
\]

Some problems go away if continuous states are discretized into cells, BUT:
- approach is no longer comprehensive
- power of continuous mathematics is lost

**New multitarget state estimators must be defined and proved optimal, consistent, etc.**!
Application 1: Poorly Characterized Data

- SAR features
- LRO features
- MTI features
- ELINT features
- SIGINT features

Interpretation by operators sent out on coded datalink alphanumerical format messages corrupted by unknowable variations ("fat-fingering", fatigue, overloading, differences in training & ability)

Systematic Bayes-rule methodology for modeling poorly characterized data
Modeling Ill-Characterized Data

- **Modeling “ambiguous data”** as random closed subsets of observation space.

<table>
<thead>
<tr>
<th>$\Sigma_{A}(f')$</th>
<th>$\Theta$</th>
<th>$\Sigma_{\Phi}(X \Rightarrow S)$</th>
<th>$\Sigma_{A}(W)$</th>
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<tbody>
<tr>
<td>vague</td>
<td>imprecise</td>
<td>contingent</td>
<td>general</td>
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<tr>
<td><em>(fuzzy)</em></td>
<td><em>(Dempster-</em>)</td>
<td><em>(rules)</em></td>
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- **Choose specific modeling technique**

- **Generalized (i.e., unnormalized) likelihood function** $\rho(\Theta|x)$

- **Practical implementation via “matching models”** $\rho(\Theta|x) = \Pr(\Theta \cap \Sigma_x \neq \emptyset)$

- **Single- or multi-target Bayes filters**

- **CAUTION!** partially heuristic

- **CAUTION!** non-Bayesian
**Generalized Likelihood of Datalink Message Line**

**SAR line from formatted datalink message**

\[
\begin{array}{cccccccccccc}
IMA/SAR/ & 22.0F/ & 29.0F/ & 14.9F/ & Y/ & -/ & Y/ & -/ & -/ & T/ & 6/ & -/
\end{array}
\]

- Convert features to fuzzy sets
- Compare to fuzzy templates in database
- Compute attribute likelihoods

\[
\rho_{SAR}(SAR|x) = \rho_1(x) \land \rho_2(x) \land \ldots \land \rho_{10}(x) \land \rho_{11}(x)
\]

\[
a \land b = \frac{ab}{a + b - ab}
\]

'\land' operation models statistical correlation intermediate between perfect correlation and independence
Application 2: Poorly Characterized Likelihoods

statistically uncharacterizable
“extended operating conditions” (EOCs)

dents
wet mud
turret articulations
non-standard equipment (e.g. hibachi welded on motor)
irregular placement of standard equipment (e.g. grenade launchers)

systematic Bayes-rule methodology for hedging against unknowable statistics
Modeling Ill-Characterized Likelihoods

\[ \ell^i_z(c) = f_i(z \mid c) \]

nominal likelihood function

\[ \ell^i_z(c) \in \Sigma^i_z(c) \]

random error bar on pixel likelihood (random interval)

\[ = \Sigma_z^1(c) \cdots \Sigma_z^N(c) \]

random error bar on image likelihood (interval arithmetic)

\[ \mu_{\hat{z},c}^i(x) = \Pr(x \in \Sigma_{\hat{z}}(c)) \]

fuzzy error bar on image likelihood
Application 3: Scientific Performance Estimation

multisensor-multitarget algorithm

multitarget miss distance

ordinary metrics

miss distance

I.D. miss

track purity

mathematical information produced by algorithm (multitarget Kullback-Leibler)
Oliver Drummond et. al. have proposed optimal truth-to-track association for evaluating multitarget tracker algorithms.

\[ d(G, X) = \min_{\pi} \frac{1}{\gamma} \sum_{i=1}^{\gamma} \| g_i - x_{\pi i} \| \]

\textbf{Wasserstein distance} rigorously and tractably generalizes intuitive approach.
Application 4: Group-Target Filtering

- **detect**
  (i.e., this target group is actually a group target)

- **classify**
  (e.g., this group target is an armored brigade)

- **track**
  (e.g., this group target is moving towards objective A)

- **group target 1**
  (armored infantry)

- **group target 2**
  (tank brigade)

- systematic Bayes foundation for group target filtering

- systematic, principled approximation strategies
Approximate 1st-Order Multi-Group Bayes Filter

- Random observation sets $Z$ produced by targets in group targets
- Random group-set $\Gamma_{k|k}$
- Multi-group motion
- State space
- Observation space

**1st-moment “group PHD” filter**

$D_{k|k}(g, x|Z^{(k)}) \rightarrow D_{k+1|k}(g, x|Z^{(k)}) \rightarrow D_{k+1|k+1}(g, x|Z^{(k+1)})$

**usually computationally hopeless!**

$\rightarrow f_{k|k}(X_k|Z^{(k)}) \rightarrow f_{k+1|k}(X_{k+1}|Z^{(k)}) \rightarrow f_{k+1|k+1}(X_{k+1}|Z^{(k+1)})$
multiple sensors on multiple platforms

- data
- attributes
- language
- rules

predicted multitarget system

sensor and platform controls to best detect, track, & identify all targets

multisensor-multitarget observation
all observations regarded as single observation

multitarget system
all targets regarded as single system (possibly undetected & low-observable)

multiple sensors on multiple platforms
all sensors on all platforms regarded as single system

Application 5: Sensor Management

- systematic Bayesian control-theoretic foundation for sensor management
- systematic, principled approximation strategies
Multi-Sensor / Target Mgmt Based on MHC’s

use multi-hypothesis correlator (MHC) as filter part of sensor mgmt
“Finite-set statistics” (FISST) provides a systematic, Bayes-probabilistic foundation and unification for multisource, multitarget data fusion

- Is an “engineering friendly” formulation of point process theory, the recognized theoretical foundation for stochastic multi-object systems
- Has led to systematic foundations for previously murky problems: e.g. group-target fusion
- Is leading to new computational approaches, e.g. PHD filter; multistep look-ahead sensor management
- Currently being investigated by other researchers:
  - Defense Sciences Technology Office (Australia), U. Melbourne (Australia), Swedish Defense Research Agency, Georgia Tech Research Institute, SAIC Corp.