Continuations and Transducer Composition

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PLDI 2006
The Big Idea

Observation
Some programs easier to write with transducer abstraction.

Goal
Design features and compilation story to support this abstraction.
The Big Idea

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Some programs easier to write with transducer abstraction.

Goal
Design features and compilation story to support this abstraction.

Oh...
Transducer ≡ Coroutine ≡ Process
A computational analogy

The world of functions

- Agents are functions.
- Functions are stateless.
- Composed with $\circ$ operator: $h = f \circ g$. 

A computational analogy

The world of functions

- Agents are functions.
- Functions are stateless.
- Composed with $\circ$ operator: $h = f \circ g$.

The world of online transducers

- Agents are input/compute/output processes.
- Processes have local, bounded state.
- Composed with Unix $|$ operator: $h = g \mid f$. 

![Diagram](image-url)
Online transducers

- DSP networks
  Convolve / integrate / filter / difference / . . .
- Network-protocol stacks ("micro-protocols", layer integration)
  packet-assembly / checksum / order / http-parse / html-lex / . . .
- Graphics processing
  viewpoint-transform / clip1 / . . . / clip6 / z-divide / light / scan
- Stream processing
- Unix pipelines
  :
Optimisation across composition

Functional paradigm

$f \circ g$ optimised by $\beta$-reduction:

\[
f = \lambda y \cdot y + 3 \\\ng = \lambda z \cdot z + 5
\]
Optimisation across composition

Functional paradigm

\( f \circ g \) optimised by \( \beta \)-reduction:

\[
\begin{align*}
  f &= \lambda y . y + 3 \\
  g &= \lambda z . z + 5 \\
  \circ &= \lambda m n . \lambda x . m(n \, x) \quad \text{("Plumbing" made explicit in \( \lambda \) rep.)}
\end{align*}
\]
Optimisation across composition

Functional paradigm

$f \circ g$ optimised by $\beta$-reduction:

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\begin{align*}
  f &= \lambda y \cdot y + 3 \\
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  \circ &= \lambda m n \cdot \lambda x . m(n x) \quad \text{("Plumbing" made explicit in } \lambda \text{ rep.)}
\end{align*}
\]

\[
f \circ g = (\lambda m n \cdot \lambda x . m(n x))(\lambda y . y + 3)(\lambda z . z + 5)
\]
Optimisation across composition

Functional paradigm

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\begin{align*}
  f &= \lambda y \cdot y + 3 \\
g &= \lambda z \cdot z + 5 \\
\circ &= \lambda m n \cdot \lambda x . m(nx) \quad \text{("Plumbing" made explicit in \( \lambda \) rep.)}
\end{align*}
\]

\[
\begin{align*}
f \circ g &= (\lambda mn . \lambda x . m(nx))(\lambda y . y + 3)(\lambda z . z + 5) \\
&= \lambda x . (\lambda y . y + 3)((\lambda z . z + 5)x) \\
&= \lambda x . (\lambda y . y + 3)(x + 5) \\
&= \lambda x . (x + 5) + 3 \\
&= \lambda x . x + (5 + 3) \\
&= \lambda x . x + 8
\end{align*}
\]
Optimisation across composition

Transducer paradigm

No good optimisation story.

Optimisation across composition is key technology supporting abstraction:
Enables construction by composition.

If only...
Optimisation across composition

Transducer paradigm
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If only . . .
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Transducer paradigm

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Optimisation across composition is key technology supporting abstraction: Enables construction by composition.

If only...
Strategy

- Build transducers from continuations.
Strategy

- Build transducers from continuations.
- Build continuations from λ.
Strategy

- Build transducers from continuations.
- Build continuations from $\lambda$.
- Handle $\lambda$ well.
Strategy

- Build transducers from continuations.
- Build continuations from $\lambda$.
- Handle $\lambda$ well.
- Watch what happens.
Tool: Continuation-passing style (CPS)

Restricted subset of $\lambda$ calculus: Function calls do not return.

Thus cannot write $f(g(x))$.

Must pass extra argument—the continuation—to each call, to represent rest of computation:

\[
(- a (* b c)) \Rightarrow (* b c (\lambda (\text{temp}) (- a \text{ temp} \text{ halt})))
\]
Tool: Continuation-passing style (CPS)

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\[-a (* b c) \Rightarrow (* b c (\lambda (temp) (- a temp halt)))\]

CPS is the “assembler” of functional languages.
CPS Payoff

CPS is universal representation of control & env.

<table>
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<tr>
<td>coroutine switch</td>
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...
Writing transducers with \texttt{put} and \texttt{get}

\begin{verbatim}
(define (send-fives)
  (put 5)
  (send-fives))
\end{verbatim}
Writing transducers with \texttt{put} and \texttt{get}

\begin{verbatim}
(define (send-fives)
  (put 5)
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(define (doubler)
  (put (* 2 (get)))
  (doubler))
\end{verbatim}
Writing transducers with \texttt{put} and \texttt{get}

\begin{verbatim}
(define (send-fives)
  (put 5)
  (send-fives))

(define (doubler)
  (put (* 2 (get)))
  (doubler))

(define (integ sum)
  (let ((next-sum (+ sum (get))))
    (put next-sum)
    (integ next-sum)))
\end{verbatim}
Tool: 3CPS & transducer pipelines

\[ f \times k u d \]
Tool: 3CPS & transducer pipelines

ExpCont: rest of this stage’s computation

 Semantic domains / Types

\[ x \in \text{Value} \]
\[ k \in \text{ExpCont} = \text{Value} \rightarrow \text{UpCont} \rightarrow \text{DownCont} \rightarrow \text{Ans} \]

(a transducer)
Tool: 3CPS & transducer pipelines

Semantic domains / Types

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\[ c \in \text{CmdCont} = \text{UpCont} \rightarrow \text{DownCont} \rightarrow \text{Ans} \ (\text{a transducer}) \]
Transducers in 3CPS

Get & put in 3CPS

\[
\begin{align*}
get x k u d &= \\
put x k u d &= \\
\end{align*}
\]

Semantic domains / Types

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x &\in \text{Value} \\
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Get & put in 3CPS

\[ get \, x \, k \, u \, d = u ( \quad ) \]
\[ put \, x \, k \, u \, d = \]

Semantic domains / Types

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Get & put in 3CPS

\[
\begin{align*}
\text{get } x \ k \ u \ d &= u (\lambda x' \ u' .) \\
\text{put } x \ k \ u \ d &= \\
\end{align*}
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Transducers in 3CPS

Get & put in 3CPS

\[ \text{get } x \ k \ u \ d = u \ (\lambda x' \ u'. \ k) \]
\[ \text{put } x \ k \ u \ d = \]

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Transducers in 3CPS

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\text{put } x & \ k & \ u & \ d = 
\end{align*}
\]

Semantic domains / Types

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\end{align*}
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Transducers in 3CPS

Get & put in 3CPS

$$\text{get } x \, k \, u \, d = u \, (\lambda \, x' \, u' . \, k \, x' \, u' \, d)$$

$$\text{put } x \, k \, u \, d =$$

Semantic domains / Types

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Transducers in 3CPS

Get & put in 3CPS

\begin{align*}
get \ x \ k \ u \ d &= u \ (\lambda \ x' \ u' . \ k \ x' \ u' \ d) \\
\text{put} \ x \ k \ u \ d &= d
\end{align*}

Semantic domains / Types

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x &\in \text{Value} \\
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\end{align*}
Transducers in 3CPS

Get & put in 3CPS

\[
\text{get } x \ k \ u \ d = u (\lambda x' \ u'. \ k \ x' \ u' \ d)
\]
\[
\text{put } x \ k \ u \ d = d \ x (\quad)(\quad)
\]

Semantic domains / Types

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x \in \text{Value}
\]
\[
k \in \text{ExpCont} = \text{Value} \rightarrow \text{UpCont} \rightarrow \text{DownCont} \rightarrow \text{Ans}
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Transducers in 3CPS

Get & put in 3CPS

\[
\begin{align*}
\text{get } x k u d &= u (\lambda x' u'. k x' u' d) \\
\text{put } x k u d &= d x (\lambda d'.) 
\end{align*}
\]

Semantic domains / Types

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Transducers in 3CPS

Get & put in 3CPS

\[
\text{get } x \ k \ u \ d = u (\lambda x' \ u'. \ k \ x' \ u' \ d)
\]

\[
\text{put } x \ k \ u \ d = d \ x (\lambda d' . \ k \ \text{unit})
\]

Semantic domains / Types

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x \in \text{Value}
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k \in \text{ExpCont} = \text{Value} \rightarrow \text{UpCont} \rightarrow \text{DownCont} \rightarrow \text{Ans}
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Transducers in 3CPS

Get & put in 3CPS

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get\ x\ k\ u\ d &= u (\lambda x'\ u'. \ k\ x'\ u'\ d) \\
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\end{align*}
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Composing transducers in 3CPS

\[ \text{compose/pull } c_1 c_2 \]

Semantic domains / Types

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Composing transducers in 3CPS

\[
\text{compose/pull } c_1 \, c_2 = \lambda u \, d .
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Semantic domains / Types

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\end{align*}
Composing transducers in 3CPS

$$compose/pull \ c_1 \ c_2 = \lambda \ u \ d . \ c_2 (\lambda \ d' . ) \ d$$

Semantic domains / Types

$$x \in Value$$

$$k \in ExpCont \ = \ Value \rightarrow UpCont \rightarrow DownCont \rightarrow Ans$$

$$u \in UpCont \ = \ DownCont \rightarrow Ans$$

$$d \in DownCont \ = \ Value \rightarrow UpCont \rightarrow Ans$$

$$c \in CmdCont \ = \ UpCont \rightarrow DownCont \rightarrow Ans$$
Composing transducers in 3CPS

\[
\text{compose/pull } c_1 \circ c_2 = \lambda u d . c_2 (\lambda d' . c_1 ) d
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Composing transducers in 3CPS

\[ \text{compose/pull } c_1 \cdot c_2 = \lambda u d \cdot c_2 (\lambda d' \cdot c_1 u) \cdot d \]

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Composing transducers in 3CPS

compose/pull \( c_1 \ c_2 = \lambda u \ d \ . \ c_2 (\lambda d' \ . \ c_1 \ u \ d') \ d \)

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\( x \in \text{Value} \)

\( k \in \text{ExpCont} \ = \text{Value} \rightarrow \text{UpCont} \rightarrow \text{DownCont} \rightarrow \text{Ans} \)

\( u \in \text{UpCont} \ = \text{DownCont} \rightarrow \text{Ans} \)

\( d \in \text{DownCont} \ = \text{Value} \rightarrow \text{UpCont} \rightarrow \text{Ans} \)

\( c \in \text{CmdCont} \ = \text{UpCont} \rightarrow \text{DownCont} \rightarrow \text{Ans} \)
Transducer data/control flow in 3CPS

\[
\text{get } x \ k \ u \ d = u \ (\lambda \ x \ u' . \ k \ x \ u' \ d)
\]

\[
\text{put } x \ k \ u \ d = d \ x \ (\lambda \ d' . \ k \ \text{unit} \ u \ d)
\]

\[
\text{compose/pull} \ c_1 \ c_2 = \lambda \ u \ d . \ c_2 (\lambda \ d' . \ c_1 \ u \ d') \ d
\]
Transducer data/control flow in 3CPS

\[
\text{get } x \ k \ u \ d = u \ (\lambda x \ u'. \ k \ x \ u' \ d)
\]

\[
\text{put } x \ k \ u \ d = d \ x \ (\lambda d'. \ k \ \text{unit} \ u \ d)
\]

compose/pull \quad c_1 \ c_2 = \lambda u \ d . \ c_2 \ (\lambda d' . \ c_1 \ u \ d') \ d

All the “plumbing” made explicit in three short equations.
A toy example

(\() \hspace{1cm} ; \text{Put-5}
(\text{letrec } ((\text{lp1 } (\lambda () \text{ (put 5) (lp1)})))
(\text{lp1}))

(\() \hspace{1cm} ; \text{Doubler}
(\text{letrec } ((\text{lp2 } (\lambda ()
\text{ (put (* 2 (get)))
(\text{lp2)}))))
(\text{lp2})))
After CPS conversion

(\( \lambda \ (k1 \ u1 \ d1) \) ; Put-5
  (letrec ((lp1 (\( \lambda \ (k1a \ u1a \ d1a) \)
    (d1a 5 (\( \lambda \ (d1b) \) (lp1 k1a u1a d1b))))))
    (lp1 k1 u1 d1)))

(\( \lambda \ (k2 \ u2 \ d2) \) ; Doubler
  (letrec ((lp2 (\( \lambda \ (k2a \ u2a \ d2a) \)
    (u2a (\( \lambda \ (x \ u2b) \)
        (d2a (* 2 x)
          (\( \lambda \ (d2b) \)
              (lp2 k2a u2b d2b))))))
    (lp2 k2 u2 d2))))
(compose/pull put-5 doubler)

((\(c1\ c2\)) ; Compose/pull
 (\(k\ u\ d\) (c2 k (\(d'\) (c1 k u d')) d)))
(\(k1\ u1\ d1\)) ; Put-5
 (letrec ((lp1 (\(k1a\ u1a\ d1a\)
 (d1a 5 (\(d1b\) (lp1 k1a u1a d1b))))))
 (lp1 k1 u1 d1)))
(\(k2\ u2\ d2\)) ; Doubler
 (letrec ((lp2 (\(k2a\ u2a\ d2a\)
 (u2a (\(x\ u2b\)
 (d2a (* 2 x)
 (\(d2b\)
 (lp2 k2a u2b d2b)))))))))
(lp2 k2 u2 d2))
Eliminate useless variables (1991)
(compose/pull put-5 doubler)

(((λ (c1 c2) ; Compose/pull
     (λ (k u d) (c2 (λ (d’) (c1 d’)) d)))
(λ (d1)) ; Put-5
    (letrec ((lp1 (λ (d1a)
                  (d1a 5 (λ (d1b) (lp1 d1b))))))
       (lp1 d1)))
(λ (u2 d2) ; Doubler
    (letrec ((lp2 (λ (u2a d2a)
                  (u2a (λ (x u2b)
                        (d2a (* 2 x)
                             (λ (d2b)
                                (lp2 u2b d2b))))))
       (lp2 u2 d2))))
(compose/pull put-5 doubler)

((λ (c1 c2) ; Compose/pull
  (λ (k u d) (c2 (λ (d’) (c1 d’)) d)))
(λ (d1)) ; Put-5
  (letrec ((lp1 (λ (d1a)
      (d1a 5 (λ (d1b) (lp1 d1b)))))
    (lp1 d1)))
(λ (u2 d2)) ; Doubler
  (letrec ((lp2 (λ (u2a d2a)
      (u2a (λ (x u2b)
        (d2a (* 2 x)
          (λ (d2b)
            (lp2 u2b d2b))))))))
    (lp2 u2 d2)))

η-reduce (1935)
(compose/pull put-5 doubler)

((\(c1 \ c2\) \n  (\(k \ u \ d\) (c2 c1 d)))
 (\(d1\) \n  (letrec ((lp1 ((lp1 (\(d1a\)\n      (d1a 5 lp1)))))
    (lp1 d1)))))
 (\(u2 \ d2\) \n  (letrec ((lp2 ((lp2 (\(u2a \ d2a\)\n      (u2a (\(x \ u2b\)\n        (d2a (* 2 x)
        (\(d2b\)
          (\(d2b\)
            (lp2 u2b d2b))))))))
    (lp2 u2 d2))))
((\(c1\ c2\) ) ; Compose/pull
  (\(k\ u\ d\) (c2 c1 d)))
(\(d1\) ) ; Put-5
  (letrec ((lp1 (\(d1a\)
    (d1a 5 lp1))))
    (lp1 d1)))
(\(u2\ d2\) ) ; Doubler
  (letrec ((lp2 (\(u2a\ d2a\)
    (u2a (\(x\ u2b\)
      (d2a (* 2 x)
        (\(d2b\)
          (lp2 u2b d2b)))))))
    (lp2 u2 d2))))

\(\beta\)-reduce whole thing (1935)
(compose/pull put-5 doubler)

(\ (k u d)
  ((\ (u2 d2) ; Doubler
      (letrec ((lp2 (\ (u2a d2a)
                     (u2a (\ (x u2b)
                           (d2a (* 2 x)
                           (\ (d2b)
                               (lp2 u2b d2b)))))))
        (lp2 u2 d2)))))

(\ (d1) ; Put-5
  (letrec ((lp1 (\ (d1a) (d1a 5 lp1)))
            (lp1 d1)))
    d))
(compose/pull put-5 doubler)

(\(k\) \(u\) \(d\))

((\(u2\) \(d2\)) ; Doubler
  (letrec ((lp2 (\(u2a\) \(d2a\))
    (u2a (\(x\) \(u2b\))
      (d2a (* 2 x)
        (\(d2b\)
          (lp2 u2b d2b))))))))

(lp2 u2 d2))

(\(d1\)) ; Put-5

(letrec ((lp1 (\(d1a\) (d1a 5 lp1))))
  (lp1 d1))

\(\beta\) again (1935)
(compose/pull put-5 doubler)

(\ (k u d)
  (letrec ((lp2 (\ (u2a d2a)
               (u2a (\ (x u2b)
                      (d2a (* 2 x)
                           (\ (d2b)
                              (lp2 u2b d2b)))))));
   (lp2 (\ (d1) ; Put-5
          (letrec ((lp1 (\ (d1a) (d1a 5 lp1))))
                 (lp1 d1))
        d)))
(compose/pull put-5 doubler)

\((\lambda (k \ u \ d)\\ (letrec ((lp2 (\lambda (u2a \ d2a)\\ (u2a (\lambda (x \ u2b)\\ (d2a (* 2 x)\\ (\lambda (d2b)\\ (lp2 u2b d2b)))))\\ (lp2 (\lambda (d1) ; Put-5\\ (letrec ((lp1 (\lambda (d1a) (d1a 5 lp1)))\\ (lp1 d1))\\ d)))))\\

Hoist inner letrec. (1980’s)
(compose/pull put-5 doubler)

(\ (k u d)
  (letrec ((lp2 (\ (u2a d2a)
                      (u2a (\ (x u2b)
                              (d2a (* 2 x)
                                (\ (d2b)
                                   (lp2 u2b d2b))))))
          (lp1 (\ (d1a) (d1a 5 lp1))))
      (lp2 (\ (d1) (lp1 d1))
           d)))
(compose/pull put-5 doubler)

(\ (k u d)
   (letrec ((lp2 (\ (u2a d2a)
                 (u2a (\ (x u2b)
                        (d2a (* 2 x)
                                (\ (d2b)
                                   (lp2 u2b d2b))))))
        (lp1 (\ (d1a) (d1a 5 lp1))))
    (lp2 (\ (d1) (lp1 d1)
            d))))

\eta\text{-reduce} (1935)
(compose/pull put-5 doubler)

(λ (k u d)
  (letrec ((lp2 (λ (u2a d2a)
      (u2a (λ (x u2b)
        (d2a (* 2 x)
        (λ (d2b)
          (lp2 u2b d2b))))))
      (lp1 (λ (d1a) (d1a 5 lp1)))
      (lp2 lp1
d)))
(compose/pull put-5 doubler)

(λ (k u d)
  (letrec ((lp2 (λ (u2a d2a)
               (u2a (λ (x u2b)
                    (d2a (* 2 x)
                         (λ (d2b)
                            (lp2 u2b d2b)))))))
               (lp1 (λ (d1a) (d1a 5 lp1))))
             (lp2 lp1 d)))

Super-β: u2a = u2b = lp1 (2006)
(compose/pull put-5 doubler)

(λ (k u d)
   (letrec ((lp2 (λ (u2a d2a)
       (lp1 (λ (x u2b)
           (d2a (* 2 x)
           (λ (d2b)
               (lp2 lp1 d2b)))))))
     (lp1 (λ (d1a) (d1a 5 lp1))))
   (lp2 lp1 d)))
(compose/pull put-5 doubler)

(\(k\ u\ d\)
  (letrec ((lp2 (\(u2a\ d2a\)
                   (lp1 (\(x\ u2b\)
                          (d2a (* 2 x)
                              (\(d2b\)
                                 (lp2 lp1 d2b))))))
         (lp1 (\(d1a\) (d1a 5 lp1)))))
    (lp2 lp1 d)))

Eliminate useless \(u2a\), \(u2b\).
(compose/pull put-5 doubler)

(\ (k u d)
  (letrec ((lp2 (\ (d2a)
    (lp1 (\ (x)
      (d2a (* 2 x)
        (\ (d2b)
          (lp2 d2b))))))
    (lp1 (\ (d1a) (d1a 5))))
  (lp2 d)))
(compose/pull put-5 doubler)

(\ (k u d)
  (letrec ((lp2 (\ (d2a)
                (lp1 (\ (x)
                     (d2a (* 2 x)
                     (\ (d2b)
                       (lp2 d2b)))))))
       (lp1 (\ (d1a) (d1a 5))))
  (lp2 d)))

$\eta$-reduce. (1935)
(compose/pull put-5 doubler)

(λ (k u d)
  (letrec ((lp2 (λ (d2a)
              (lp1 (λ (x)
                     (d2a (* 2 x) lp2)))))
          (lp1 (λ (d1a) (d1a 5))))
  (lp2 d)))
(compose/pull put-5 doubler)

(\k\u\d
  (letrec ((lp2 (\d2a
    (lp1 (\x
      (d2a (* 2 x) lp2)))))
    (lp1 (\d1a (d1a 5))))
  (lp2 d)))

Inline & \(\beta\)-reduce \(\textnormal{lp1}\) application. (1935)
(compose/pull put-5 doubler)

(λ (k u d)
  (letrec ((lp2 (λ (d2a)
      ((λ (d1a) (d1a 5))
        (λ (x) (d2a (* 2 x) lp2))))))
   (lp2 d)))
Two more $\beta$ steps. (1935)
Liftoff!
Issues

- Linear “pipeline” topology wired in. Can we generalise?
- Can it be typed?
- OK, it works “by hand.” Can it be implemented?
Issues

- Linear “pipeline” topology wired in. Can we generalise?
- Can it be typed?
- OK, it works “by hand.” Can it be implemented?

Yes.
Channels in CPS

Explicit channels permit non-linear control/data-flow topologies.

Same optimisation story applies as in 3CPS case.
Types for functional coroutines

\((\alpha, \beta)\)Channel /* coroutine connection: send an \(\alpha\), get a \(\beta\). */

\[
\text{switch} : \alpha \times (\alpha, \beta)\text{Channel} \rightarrow \beta \times (\alpha, \beta)\text{Channel}
\]

```plaintext
datatype (\alpha, \beta)\text{Channel} =
    Chan of (\alpha \times (\beta, \alpha)\text{Channel})\text{cont};

fun switch(x, Chan k) =
    callcc (fn k' => throw k (x, Chan k'));
```

Details are in the paper.
Composing non-iterative computations

Some producers are truly recursive:

(define (gen-fringe tree chan)
  (if (leaf? tree)
      (put (leaf:val tree) chan)
      (let ((chan (gen-fringe (tree:left tree) chan)))
        (gen-fringe (tree:right tree) chan))))

What if we compose with summing consumer?
Composing non-iterative computations

Some producers are truly recursive:

```scheme
(define (gen-fringe tree chan)
  (if (leaf? tree)
      (put (leaf:val tree) chan)
      (let ((chan (gen-fringe (tree:left tree) chan)))
        (gen-fringe (tree:right tree) chan))))
```

What if we compose with summing consumer?

Prototype compiler produces recursive, tree-walk summation.
Experience

- Built prototype compiler for toy dialect of Scheme.
  - Direct-style front end
  - Includes call/cc
  - Standard optimisations ($\beta$, $\eta$, ...)
  - Plus $\Delta$CFA (POPL 2006), abstract GC, abstract counting ($\Gamma$CFA, ICFP 2006)

- Used for testing out Ph.D. analyses/optimisations
  Nothing transducer/coroutine specific—just a machine for attacking CPS.

- Successfully fuses put5/doubler, integrators,
  (rendered with coroutines/channels)

- Limiting reagent: Super-$\beta$. 
Related work

Transducer fusion

- Deforestation
- Haskell’s fold/build, unfold/destroy, etc.
- Clu loop generators
- APL
- Filter fusion / Integrated layer processing
Final thoughts

- It’s all about the representation.
  - $\lambda$ as essential control/env/data-structure
  - CPS $\Rightarrow$ Our *main* concern
    becomes our *only* concern.

Once in CPS, generic optimisations suffice.
Final thoughts

- It’s all about the representation.
  - \( \lambda \) as essential control/env/data-structure
  - CPS \( \Rightarrow \) Our *main* concern
    becomes our *only* concern.

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This generalises to exotic control structures.
Final thoughts

- It’s all about the representation.
  - λ as essential control/env/data-structure
  - CPS ⇒ Our *main* concern
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- Coroutines are the neglected control structure.
Final thoughts

- It’s all about the representation.
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Once in CPS, generic optimisations suffice. This generalises to exotic control structures.

- Coroutines are the neglected control structure.
- Coroutines don’t have to be heavyweight.
  ($\lambda$, CPS & static analysis are answer to efficiency issues.)
Final thoughts

- It’s all about the representation.
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- Coroutines are the neglected control structure.
- Coroutines don’t have to be heavyweight.
  ($\lambda$, CPS & static analysis are answer to efficiency issues.)

- Lots to do! (Stay tuned)
  - Full-blown SML compiler
  - TCP/IP (Foxnet)
  - DSP libs.
Thank you.