INFORMATION ASYMMETRIES IN COMMON VALUE AUCTIONS WITH DISCRETE SIGNALS

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Ad Auctions and Information Asymmetries

Web Impression

hulu

NETFLIX
Ad Auctions and Information Asymmetries

Web Impression

Amazon Cookie $x$

Kayak Cookie $y$
Ad Auctions and Information Asymmetries

Web Impression of Unknown Common Value $V$

Amazon Cookie $x$

Kayak Cookie $y$
Common Value Single-Item Auction

Item of Unknown Common Value $V$

Private Signal $x$

Private Signal $y$
Assumptions

- Signal of each bidder comes from Discrete Ordered Set
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- **Informative**: Higher Signal – Higher Expected Value
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- **Affiliated:** Higher Signal – Stochastically Higher Signal for Opponent
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- **Informative**: Higher Signal – Higher Expected Value

- **Affiliated**: Higher Signal – Stochastically Higher Signal for Opponent

- **Signals Drawn from Arbitrary Asymmetric and Correlated Distribution (with full support)**
Traditional Applications

Oil Lease for land with unknown common value $V$

Access to Test $x$

Access to Test $y$
Main Questions

- How do bidders behave in equilibrium?
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• Which auction formats yield higher revenue?
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- How do bidders behave in equilibrium?
- Which auction formats yield higher revenue?
- How does extra information affect player utilities and seller’s revenue?
Auctions Considered – Hybrid Auctions

- Highest Bidder Wins.

- Pays his bid with some positive probability $\kappa$ and the second highest bid with the remaining
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- Pays his bid with some positive probability \( \kappa \) and the second highest bid with the remaining
- \( \kappa = 1 \): First Price Auction
Auctions Considered – Hybrid Auctions

- Highest Bidder Wins.

- Pays his bid with some positive probability $\kappa$ and the second highest bid with the remaining

  - $\kappa = 1$: First Price Auction

  - $\kappa \to 0$: Limit Equilibrium of Second Price Auction (Equilibrium Selection)
Related Work

- Value of information in auctions: [Milgrom ’79], [Milgrom/Weber ’82], . . .
- Common-value auctions with binary signals: [Banerjee ’05], [Abraham/Athey/Babaioff/Grubb ’12]
- Continuous values/signals: [Engelbrecht-Wiggans/Milgrom/Weber ’83], . . ., [Parreiras ’06]
- Other common-value models: [Rothkopf ’69], [Reece ’78], [Hausch ’87], [Wang ’91], [Laskowski/Slonim ’99], [Kagel/Levin ’02].
- Value of information: [Lehmann ’88], [Persico ’00], [Athey/Levin ’01], [Compte/Jehiel’07], [Es˝o/Szentes ’07]
How do bidders behave in equilibrium?

Theorem. There exists a unique equilibrium which is mixed and can be found constructively.
How do bidders behave in equilibrium?

- Mixed Nash Equilibrium must look as follows:
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\[ E_V(1,1) \]

\[ b_1 b_2 b_3 b_4 b_5 \]

Range of Bids Conditional on Receiving Signal 3

Player X

1 2 3 4 5

Player Y

1 2 3 4
How do bidders behave in equilibrium?

- Mixed Nash Equilibrium must look as follows:

```
\( E \)  \( V \)
1  1  
2  3  4  5

Range of Bids Conditional on Receiving Signal 3
```

<table>
<thead>
<tr>
<th>Player X</th>
<th>Player Y</th>
</tr>
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<tbody>
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CDFs of Player X

CDFs of Player Y

\( EV(1,1) \)

\( \bar{b} \)
Unique Equilibrium

- There exists a unique equilibrium defined by a recursive process:

\[
E_{V}(1,1) \]

CDFs of Player X

CDFs of Player Y

Player Y

Player Z

\(b\)
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![Diagram showing CDFs of players X, Y, and Z with bids and EV(1,1) and \( \bar{b} \).]
Unique Equilibrium

- There exists a unique equilibrium defined by a recursive process:

![Diagram showing CDFs of Player X and Player Y with bids on the x-axis and EV(1,1) on the y-axis. Initial bids are labeled 1, 2, 3, 4, and 5. CDFs for Player Z are also shown.]
Unique Equilibrium

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Unique Equilibrium

- As $\kappa$ approaches 0 (Second Price Auction)

CDFs of Player $X$

CDFs of Player $Y$

$EV(1,1)$ $EV(1,2)$ ... $EV(3,3)$ ... $EV(5,4)$

Player $Y$

Player $Z$
A Simple Example: First Price – Binary Signal

- One player receives a binary signal and the other is uninformed

\[ F_L^Y(b) \]

Player Y

\[ F_H^Y(b) \]

\[ F_Z^Y(b) \]

Player Z

\[ E[V|L] \quad E[V] \quad E[V|H] \]
A Simple Example: First Price – Binary Signal

\[ F^Z(b)(E[V|H] - b) = E[V|H] - E[V] \]

\[ \Pr[H] F^Y_H(b)(E[V|H] - b) + \Pr[L] (E[V|L] - b) = 0 \]
A Simple Example: First Price – Binary Signal

- One player receives a binary signal and the other is uninformed

\[
F^Y_H(b) = \frac{\Pr[L] (b - E[V|L])}{\Pr[H] (E[V|H] - b)}
\]

\[
F^Z(b) = \frac{E[V|H] - E[V]}{E[V|H] - b}
\]
A Simple Example: Limit to Second Price

- One player receives a binary signal and the other is uninformed.
A Simple Example: Limit to Second Price

- One player receives a binary signal and the other is uninformed
Second Price Selection – No Revenue Collapse

- Different prediction than the collapsed revenue equilibrium predicted by tremble-robust equilibrium selection of Abraham et al.
Only one informed bidder

- Informed Bidder bids “truthfully”
- Uninformed Bidder simulates informed bidder’s bid
- First and Second Price: Revenue Equivalent
Complete Revenue Ranking

• **Our Result:** The equilibrium revenue is a non-increasing function of the probability $\kappa$ that the winner pays his bid.
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  - Revenue monotonically increases as we move from first price to second price
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- **Our Result:** The equilibrium revenue is a non-increasing function of the probability $\kappa$ that the winner pays his bid.

- **Complete Revenue Ranking among Hybrid Auctions**
  - First Price – Worst Revenue
  - Revenue monotonically increases as we move from first price to second price
  - Limit Equilibrium of Second Price Selected, has highest revenue among hybrid auctions
Should seller reveal his private signals?

Web Site Visitor of Unknown Common Value $V$

Amazon Cookie $x$

MSN Cookie $z$

Kayak Cookie $y$
Failure of the Linkage Principle

- **Linkage Principle [Milgrom-Weber’82]:** In common value settings, the more information you link to the price of the winning bidder the higher the revenue.
- **Implication:** Seller should always reveal affiliated signals.
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  - Breaks even in first price auction when each bidder and the auctioneer have binary signals of different accuracy.
Failure of the Linkage Principle

- First Price Auction
- Value either 0 or 1, a prior is 1 with prob. $a$
- Player $Y$ gets a binary signal that is correct with $p_Y$
- Player $Z$ gets a binary signal that is correct with $p_Z$
- Seller has a signal that is correct with $q$

$U_Y + U_Z$: without revelation
$U_Y + U_Z$: with revelation

$p_Y = 0.9, p_Z = 0.75, q = 0.7$
How does extra information affect player utilities?

Third Party Information Sellers

Buy Access to Extra Cookies

Kayak Cookie $z$

Amazon Cookie $y$

Ad Space
Surprising Externality Effects

- Obviously, information can have negative externalities
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But...

- Information can also have positive externalities
Surprising Externality Effects

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But…

- Information can also have positive externalities
  - E.g. both bidders might strictly prefer that a specific bidder receives the extra signal
Recap

• **Information Asymmetries** in Common Value Auctions

• **Unique Equilibrium** if winner pays his bid with positive probability

• **Failure of the Linkage Principle** – Not always optimal for seller to reveal information even in pure common value

• **Complete Revenue Ranking**
  - Limit Equilibrium of Second Price $\geq$ Hybrid $\geq$ First Price

• Extra Information can have **positive externalities**