

Strategic Network Formation

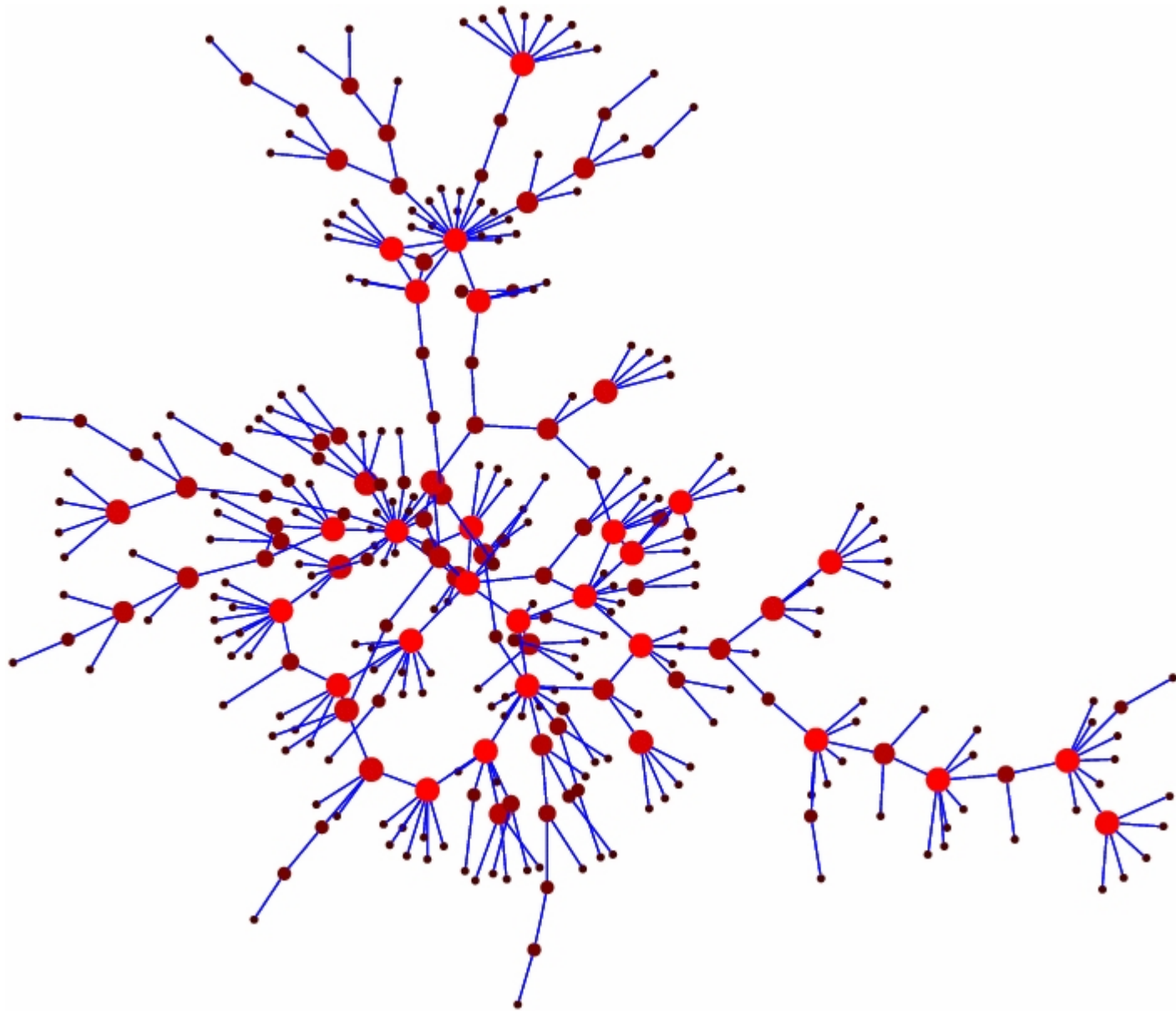
Sanjeev Goyal
Cambridge University

Outline

- The structure of networks
- The formation of networks
- Network Advantages
- Good and bad networks.
- Interesting open problems

1. The structure of networks

- A network describes a collection of nodes and the links between them.
- Once you begin to study networks it is difficult not to see them everywhere.
- Examples include: Internet, World wide web, airline networks, friendships, research alliances, co-authorships, trade and exchange.



1. Structure of networks

- Three key properties: the **degree distribution**, the **clustering**, the **average distance**.
- **Degree distribution:** Average degree is very small compared to number of nodes and there is enormous inequality. On the web, in a study from 2000, there were over 200 million web sites, average degree only 7.5, most web sites less than 10 links, but some have thousands of links!
- **Clustering:** Clustering is very high in social networks. If links are formed at random then in a large n -node network with average degree k the clustering would be roughly k/n . In the economics co-author network the clustering coefficient was 0.157; over 7,000 times the level in a random network.
- **Average distances:** The average distance between nodes is very small. In the world wide web, the giant component comprised about 180 million web sites and the average distance was only 6. Similarly, in the firm network there are over 4000 nodes, but the average distance was around 4.

1. Structure of networks

- **Small worlds:** A network with small average degrees, high clustering, and small average distances has been called a *small world* network by Watts and Strogatz (1998). The expression is originally due to Milgram (1967).
- Social and economic networks display common features: low average degree, very unequal degree distribution, clustering is high and the average distance between nodes is small. Thus star network and its variants -- such as inter-linked stars and core-periphery networks -- capture essential elements of empirical networks.
- **Key questions:** Who forms the networks? when do they have this structure? Why does it matter?

2. Formation of networks

- Important feature in many contexts:
 1. **Externality/spillover**: A link between A and B affects the payoffs of C as well as the rewards for C from linking with A and B.
 2. **Linking is a decision**: Firms can choose whether or not to form an alliance; I can decide whether or not to form a hyperlink with a web-site. You and I decide on whether or not to collaborate in research

Combine 1 and 2: linking is a strategic activity and so we study **Games of Network Formation**.

2. Formation of networks

- Key issues in modelling:
- Payoffs: linking generates payoffs; how to allocate the payoffs.
- Decision power: who decides on the link, one person, two persons, all players etc.
- Information: what do I know when I form a link?
- We start with the simplest case: a player decides on whether to link with others unilaterally. No transfers or bargaining. Full Information.

2. Formation of networks

- **A simple game of network formation**
- There are a large number of players, n .
- Each player can choose to link with any subset of others. A link costs c .
- A link between A and B gives A access to information which B has on her own, as well as information which she accesses via her links, in due course.
- The payoff to a player is the information she accesses **less** the costs of links she forms.
- Payoffs to player = #[people accessed] – #[links].c
- What is the network that is formed?

2. Formation of networks

- How do we solve this game?
- ***Nash equilibrium***: specifies a strategy for each player, with the feature that every player is doing as well as possible, given what others are doing.
- What are the Nash equilibria of the game of network formation?

2. Formation of networks

- **Theorem 1**

The star is the unique equilibrium network architecture if costs of linking are not too high. [Bala and Goyal, 2000. Hojman and Szeidl, 2007, Ferri, 2007].

Key points in proof:

1. In a star network, a player is accessing everyone with just one link, and everyone is close by. So it is optimal not to form additional links.
2. a switching argument by symmetry. Take any two end-players in a tree network, they have an incentive to get closer to the centre, if long paths are bad for information transmission. A player will delete link and switch to someone closer to the centre.

2. Formation of networks

- Dynamics: suppose players can observe the network and revise links. We ask: Starting from any arbitrary network will the network converge and what will be the limit network?
- **Theorem 2**
The dynamics converge to the star network.
[Bala and Goyal, 2000; Ferri 2007].
- The star network is thus very robust and dynamically stable.
- **General message:** unequal degrees, short average distances is a property of equilibrium networks.

2. Formation of networks

- Firms, Networks and Markets

Leading firms in hi-tech industries rely on a combination of in-house and collaborative research to stay ahead.

- Strategic alliances in the face of competition.

---- an alliance improves competitive position of partners

---- it also alters incentives of other firms to initiate costly new alliances.

2. Formation of networks

A game of research network formation

- Firms bilaterally choose research links.
- Partners share technological information which lower costs of production. More links firms lead to lower costs, which leads to larger market share.
- However each link involves a fixed cost C .
- Key feature:
 1. link decided bilaterally.
 2. Networks and markets

2. Formation of networks

- How do we solve such games? Nash equilibrium is too weak as there is a coordination problem in bilateral link formation, firm A offers no link since it expects no one else to form any links.
- To avoid this problem: refine Nash equilibrium with the requirement that no two players should have an incentive to form an additional link.
- Call this solution concept ***Pairwise Nash Equilibrium***

2. Formation of networks

- Key incentive issues:
 1. starting from a network, should a firm form an additional link? This depends on a comparison of additional profits and costs of a link: whether returns are concave or convex in own links?
 2. How does others linking alter incentives of A and B: whether links are strategic complements or substitutes?
- In the classical Cournot model of oligopoly payoffs are convex in own links and links are strategic substitutes.
- What are the Pairwise Nash Equilibrium networks?

2. Formation of networks

- **Theorem 3**

A pairwise equilibrium network has the dominant group architecture.

- **Theorem 4**

If firms can make transfers for link formation then the star network and variants of the star are pairwise equilibrium networks.

[Goyal and Joshi 2003, Deorian, 2006]

- **General Message:** in a classical textbook model of oligopolistic competition, strategic networking gives rise to unequal degrees, small average distance, and high clustering in firm alliance networks.

3. Network advantages

- Does network degree and location confer advantages?
- In the communication game: network formation leads to star network in which the central hub player will have privileged access to information. In the presence of decay the spokes pay for the links, and so hub gains both ways.
- In the firm network game: unequal degrees arise in equilibrium and more links translate into higher net profits.
- **General message:** *strategic networking can create large inequalities across players who were ex-ante identical.*

4. Good and bad networks

- Key idea: links are motivated by individual incentives. Individual linking generates externalities and spillovers. So there is a tension between equilibrium and socially desirable networks.
- In communication network game: individual linking creates **positive** spillovers for others, and so equilibrium networks may be under-connected, relative to what is socially optimal.
- In the firms research networks: linking creates **negative** profit effects for other firms, so there is a pressure toward too many links in equilibrium.

[Bala and Goyal 2000; Goyal and Joshi 2003. Yi, 1998]

5. Where are we now?

- The theory of network formation is concerned with understanding how networks arise out of strategic choices of players concerning link formation.
- The theory generates surprisingly sharp predictions on equilibrium networks: unequal degrees, small average distance, but little clustering.
- Strategic networking has powerful effects on payoff inequality as well as aggregate social welfare.
- Suggests role of policy – taxes and subsidies – to reorient network formation. [Taken up by the theory of mechanism design.]

5. Where are we now?

- Theory predictions match with key aspects of empirical networks. Given the simplicity of the theory models there is hope that further development will enable a closer match between theory and empirics. Some directions for further work:
 1. Models of weighted networks
 2. Dynamic Network formation, (as are models in CS and physics).
 3. Real world networks are complicated and players have very incomplete information on them

CONNECTIONS

An
Introduction
to the
Economics
of Networks

SANJEEV GOYAL

