The PlusCal Code for Byzantizing Paxos by Refinement

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This document contains the PlusCal code of the four specifications described in the paper Byzantizing Paxos by Refinement, which is available on the Web [1]. Comments are indicated by a gray background. The code comes from the TLA⁺ modules, which are available on the Web site. The algorithms use some constants that are declared in the TLA⁺ modules, but the meanings of those constants should be clear. The code was formatted by hand, so errors could have been introduced.

1 Algorithm Consensus

We specify the safety property of consensus as a trivial algorithm that describes the allowed behaviors of a consensus algorithm. It uses the variable *chosen* to represent the set of all chosen values. The algorithm allows only behaviors that contain a single state-change in which the variable *chosen* is changed from its initial value $\{\}$ to the value $\{v\}$ for an arbitrary value v in Value. The algorithm itself does not specify any fairness properties, so it also allows a behavior in which *chosen* is not changed. We could use a translator option to have the translation include a fairness requirement, but we don't bother because it is easy enough to add it by hand to the safety specification that the translator produces.

A real specification of consensus would also include additional variables and actions. In particular, it would have *Propose* actions in which clients propose values and *Learn* actions in which clients learn what value has been chosen. It would allow only a proposed value to be chosen. However, the interesting part of a consensus algorithm is the choosing of a single value. We therefore restrict our attention to that aspect of consensus algorithms. In practice, given the algorithm for choosing a value, it is obvious how to implement the *Propose* and *Learn* actions.

For convenience, we define the macro Choose() that describes the action of changing the value of chosen from $\{\}$ to $\{v\}$, for a nondeterministically chosen v in the set Value. (There is little reason to encapsulate such a simple action in a macro; however our other algorithms are easier to read when written with such macros, so we start using them now.) The **when** statement can be executed only when its condition, $chosen = \{\}$, is true. Hence, at most one Choose() action can be performed in any execution. The **with** statement executes its body for a nondeterministically chosen v in Value. Execution of this statement is enabled only if Value is non-empty-something we do not assume at this point because it is not required for the safety part of consensus, which is satisfied if no value is chosen.

We put the Choose() action inside a **while** statement that loops forever. Of course, only a single Choose() action can be executed. The algorithm stops after executing a Choose() action. Technically, the algorithm deadlocks after executing a Choose() action because control is at a statement whose execution is never enabled. Formally, termination is simply deadlock that we want to happen. We could just as well have omitted the **while** and let the algorithm terminate. However, adding the **while** loop makes the TLA⁺ representation of the algorithm a tiny bit simpler.

```
--algorithm Consensus {
    variable chosen = \{\};
    macro Choose() { when chosen = \{\};
        with (v \in Value) { chosen := \{v\} }
    }
    { lbl: while (TRUE){ Choose() }
}
```

2 Algorithm Voting

In the algorithm, each acceptor can cast one or more votes, where each vote cast by an acceptor has the form $\langle b, v \rangle$ indicating that the acceptor has voted for value v in ballot b. A value is chosen if a quorum of acceptors have voted for it in the same ballot.

The algorithm uses two variables, *votes* and *maxBal*, both arrays indexed by acceptor. Their meanings are:

```
votes[a] The set of votes cast by acceptor a.
```

maxBal[a] The number of the highest-numbered ballot in which a has cast a vote, or -1 if it has not yet voted.

The algorithm does not let acceptor a vote in any ballot less than maxBal[a].

We specify our algorithm by the following PlusCal code. The specification *Spec* defined by this algorithm describes only the safety properties of the algorithm. In other words, it specifies what steps the algorithm may take. It does not require that any (non-stuttering) steps be taken. Liveness is discussed in the TLA⁺ specification.

```
--algorithm Voting \{
variables votes = [a \in Acceptor \mapsto \{\}],
maxBal = [a \in Acceptor \mapsto -1];
define \{
```

The **define** section adds TLA⁺ definitions of operators that can use the algorithm's variables and can be used within the algorithm.

We now define the operator SafeAt so SafeAt(b,v) is a function of the state that equals TRUE if no value other than v has been chosen or can ever be chosen in the future (because the values of the variables votes and maxBal are such that the algorithm does not allow enough acceptors to vote for it). We say that value v is safe at ballot number b iff Safe(b,v) is true. We define Safe in terms of the following two operators.

Note: This definition is weaker than would be necessary to allow a refinement of ordinary Paxos consensus, since it allows different quorums to "cooperate" in determining safety at b. This is used in algorithms like Vertical Paxos that are designed to allow reconfiguration within a single consensus instance, but not in ordinary Paxos. See [2].

We define SafeAt in terms of the following two operators.

```
VotedFor(a, b, v) \stackrel{\Delta}{=} \langle b, v \rangle \in votes[a]
True iff acceptor a has voted for v in ballot b.
```

```
DidNotVoteIn(a, b) \stackrel{\Delta}{=} \forall v \in Value : \neg VotedFor(a, b, v)
```

We now define SafeAt. We define it recursively. The nicest definition is:

```
RECURSIVE SafeAt(\_,\_)
SafeAt(b, v) \triangleq \\ \forall b = 0 \\ \forall \exists Q \in Quorum : \\ \land \forall a \in Q : maxBal[a] \geq b \\ \land \exists c \in -1 ... (b-1) : \\ \land (c \neq -1) \Rightarrow \land SafeAt(c, v) \\ \land \forall a \in Q : \\ \forall w \in Value : \\ VotedFor(a, c, w) \Rightarrow (w = v) \\ \land \forall d \in (c+1) ... (b-1), a \in Q : DidNotVoteIn(a, d)
```

However, TLAPS does not currently support recursive operator definitions. We therefore define it as follows using a recursive function definition.

```
SafeAt(b, v) \triangleq \\ \text{LET } SA[bb \in Ballot] \triangleq \\ \text{This recursively defines } SA[bb] \text{ to equal } SafeAt(bb, v). \\ \lor bb = 0 \\ \lor \exists \ Q \in \ Quorum : \\ \land \forall \ a \in \ Q : maxBal[a] \geq bb \\ \land \exists \ c \in -1 \dots (bb-1) : \\ \land \ (c \neq -1) \Rightarrow \land SA[c] \\ \land \ \forall \ a \in \ Q : \\ \forall \ w \in \ Value : \\ VotedFor(a, \ c, \ w) \Rightarrow (w = v) \\ \land \ \forall \ d \in (c+1) \dots (bb-1), \ a \in \ Q : DidNotVoteIn(a, \ d) \\ \text{IN } SA[b] \\ \}
```

There are two possible actions that an acceptor can perform, each defined by a macro. In these macros, self is the acceptor that is to perform the action. The first action, IncreaseMaxBal(b) allows acceptor self to set maxBal[self] to b if b is greater than the current value of maxBal[self].

```
\begin{array}{ll} \mathbf{macro} & IncreaseMaxBal(b) \ \{\\ \mathbf{when} & b > maxBal[self] \ ; \\ & maxBal[self] \ := \ b \\ \} \end{array}
```

Action VoteFor(b, v) allows acceptor self to vote for value v in ballot b if its **when** condition is satisfied.

```
 \begin{array}{ll} \mathbf{macro} & \mathit{VoteFor}(b,v) \ \{ \\ \mathbf{when} & \land \mathit{maxBal}[\mathit{self}] \leq b \\ & \land \mathit{DidNotVoteIn}(\mathit{self},b) \\ & \land \forall p \in \mathit{Acceptor} \backslash \{\mathit{self}\} : \\ & \forall w \in \mathit{Value} : \mathit{VotedFor}(p,b,w) \Rightarrow (w=v) \\ & \land \mathit{SafeAt}(b,v) \ ; \\ & \mathit{votes}[\mathit{self}] := \mathit{votes}[\mathit{self}] \cup \{\langle b,v \rangle\} \ ; \\ & \mathit{maxBal}[\mathit{self}] := b \\ & \} \end{array}
```

The following process declaration asserts that every process self in the set Acceptor executes its body, which loops forever nondeterministically choosing a Ballot b and executing either an IncreaseMaxBal(b) action or nondeterministically choosing a value v and executing a VoteFor(b, v) action. The single label indicates that an entire execution of the body of the **while** loop is performed as a single atomic action.

From this intuitive description of the process declaration, one might think that a process could be deadlocked by choosing a ballot b in which neither an IncreaseMaxBal(b) action nor any VoteFor(b, v) action is enabled. An examination of the TLA⁺ translation (and an elementary knowledge of the meaning of existential quantification) shows that this is not the case. You can think of all possible choices of b and of v being examined simultaneously, and one of the choices for which a step is possible being made.

```
\begin{array}{l} \mathbf{process} \; (acceptor \in Acceptor) \; \{ \\ \quad acc : \; \mathbf{while} \; (\texttt{TRUE}) \; \{ \\ \quad \quad \mathbf{with} \; (b \in Ballot) \; \{ \\ \quad \quad \mathbf{either} \; \; IncreaseMaxBal(b) \\ \quad \quad \mathbf{or} \quad \quad \mathbf{with} \; (v \in Value) \; \{ \; VoteFor(b,v) \; \} \\ \quad \quad \quad \} \\ \quad \quad \quad \} \\ \quad \quad \} \\ \quad \quad \} \\ \quad \quad \} \\ \quad \} \\ \quad \} \\ \quad \} \end{array}
```

3 Algorithm PCon

The algorithm is easiest to understand in terms of the set msgs of all messages that have ever been sent. A more accurate model would use one or more variables to represent the messages actually in transit, and it would include actions representing message loss and duplication as well as message receipt.

For our purposes, there is no need to model message loss explicitly. The safety part of the spec says only what messages may be received and does not assert that any message actually is received. Thus, there is no difference between a lost message and one that is never received. The liveness property of the spec will make it clear what messages must be received (and hence either not lost or successfully retransmitted if lost) to guarantee progress.

Another advantage of maintaining the set of all messages that have ever been sent is that it allows us to define the state function *votes* that implements the variable of the same name in the voting algorithm without having to introduce a history variable.

In addition to the variable msgs, the algorithm uses four variables whose values are arrays indexed by acceptor, where for any acceptor a:

- maxBal[a] The largest ballot number in which a has participated
- maxVBal[a] The largest ballot number in which a has voted, or -1 if it has never voted.
- maxVVal[a] If a has voted, then this is the value it voted for in ballot maxVBal; otherwise it equals None.

As in the voting algorithm, an execution of the algorithm consists of an execution of zero or more ballots. Different ballots may be in progress concurrently, and ballots may not complete (and need not even start). A ballot b consists of the following actions (which need not all occur in the indicated order).

- Phase1a The leader sends a 1a message for ballot b.
- Phase1b If maxBal[a] < b, an acceptor a responds to the 1a message by setting maxBal[a] to b and sending a 1b message to the leader containing the values of maxVBal[a] and maxVVal[a].
- Phase1c When the leader has received ballot- $b\ 1b$ messages from a quorum, it determines some set of values that are safe at b and sends 1c messages for them.
- Phase2a The leader sends a 2a message for some value for which it has already sent a ballot-b 1c message.
- Phase2b Upon receipt of the 2a message, if $maxBal[a] \leq b$, an acceptor a sets maxBal[a] and maxVBal[a] to b, sets maxVVal[a] to the value in the 2a message, and votes for that value in ballot b by sending the appropriate 2b message.

```
--algorithm PCon {
   variables maxBal = [a \in Acceptor \mapsto -1],
                maxVBal = [a \in Acceptor \mapsto -1],
                maxVVal = [a \in Acceptor \mapsto None],
                msqs = \{\}
   define {
    sentMsgs(t, b) \triangleq \{m \in msgs : (m.type = t) \land (m.bal = b)\}
    We define ShowsSafeAt so that ShowsSafeAt(Q, b, v) is true for a quorum Q iff msgs
    contain ballot-b 1b messages from the acceptors in Q showing that v is safe at b.
    ShowsSafeAt(Q, b, v) \triangleq
      LET Q1b \triangleq \{m \in sentMsgs("1b", b) : m.acc \in Q\}
              \land \, \forall \, a \in \mathit{Q} : \exists \, m \in \mathit{Q1b} : m.\mathit{acc} = a
              \land \lor \forall m \in Q1b : m.mbal = -1
                 \vee \exists m1c \in msqs:
                       \land m1c = [type \mapsto "1c", bal \mapsto m1c.bal, val \mapsto v]
                       \land \forall m \in Q1b : \land m1c.bal \geq m.mbal
                                          \land (m1c.bal = m.mbal) \Rightarrow (m.mval = v)
   }
```

The Actions

As before, we describe each action as a macro. The leader for process self can execute a Phase1a() action, which sends the ballot self 1a message.

```
macro Phase1a() { msgs := msgs \cup \{ [type \mapsto "1a", bal \mapsto self] \} \}
```

Acceptor self can perform a Phase1b(b) action, which is enabled iff b > maxBal[self]. The action sets maxBal[self] to b and sends a phase 1b message to the leader containing the values of maxVBal[self] and maxVVal[self].

```
\label{eq:macro-Phase1b(b) { when } (b > maxBal[self]) \land (sentMsgs("1a", b) \neq \{\}) ; \\ maxBal[self] := b ; \\ msgs := msgs \cup \{ [type \mapsto "1b", \ acc \mapsto self, \ bal \mapsto b, \\ mbal \mapsto maxVBal[self], \ mval \mapsto maxVVal[self]] \} ; \\ \}
```

The ballot self leader can perform a Phase1c(S) action, which sends a set S of 1c messages indicating that the value in the val field of each of them is safe at ballot b. In practice, S will either contain a single message, or else will have a message for each possible value, indicating that all values are safe. In the first case, the leader will immediately send a 2a message with the value contained in that single message. (Both logical messages will be sent in the same physical message.) In the latter case, the leader is informing the acceptors that all values are safe. (All those logical messages will, of course, be encoded in a single physical message.)

```
\begin{array}{l} \mathbf{macro}\ Phase1c(S)\ \{\\ \mathbf{when}\ \forall v\in S: \exists Q\in Quorum: ShowsSafeAt(Q,self,v);\\ msgs:=msgs\cup \{ [type\mapsto \text{``1c''},\ bal\mapsto self,\ val\mapsto v]: v\in S\, \}\\ \} \end{array}
```

The ballot self leader can perform a Phase2a(v) action, sending a 2a message for value v, if it has not already sent a 2a message (for this ballot) and it has sent a ballot self 1c message with val field v.

```
\label{eq:macro-Phase2a} \begin{split} \mathbf{macro} \ Phase2a(v) \ \{ \\ \mathbf{when} \ \land sentMsgs(\text{``2a''}, self) = \{ \} \\  \  \  \  \  \land [type \mapsto \text{``1c''}, \ bal \mapsto self, \ val \mapsto v] \in msgs \ ; \\ msgs \ := msgs \ \cup \ \{ [type \mapsto \text{``2a''}, \ bal \mapsto self, \ val \mapsto v] \ \} \\ \} \end{split}
```

The Phase2b(b) action is executed by acceptor self in response to a ballot-b 2a message. Note this action can be executed multiple times by the acceptor, but after the first one, all subsequent executions are stuttering steps that do not change the value of any variable.

```
macro Phase2b(b) {
    when b \ge maxBal[self] ;
    with (m \in sentMsgs("2a", b)) {
        maxBal[self] := b ;
        maxVBal[self] := b ;
        maxVVal[self] := m.val ;
        msgs := msgs \cup \{ [type \mapsto "2b", acc \mapsto self, bal \mapsto b, val \mapsto m.val] \} }
}
```

An acceptor performs the body of its **while** loop as a single atomic action by nondeterministically choosing a ballot in which its Phase1b or Phase2b action is enabled and executing that enabled action. If no such action is enabled, the acceptor does nothing.

```
 \begin{array}{l} \mathbf{process} \ (acceptor \in Acceptor) \ \{ \\ acc: \ \mathbf{while} \ (\mathtt{TRUE}) \ \{ \\ \quad \mathbf{with} \ (b \in Ballot) \ \{ \ \mathbf{either} \ Phase1b(b) \ \mathbf{or} \ Phase2b(b) \ \} \\ \quad \} \\ \} \\ \end{aligned}
```

The leader of a ballot nondeterministically chooses one of its actions that is enabled (and the argument for which it is enabled) and performs it atomically. It does nothing if none of its actions is enabled.

```
\begin{array}{ll} \mathbf{process} \; (leader \in Ballot) \; \{ \\ & ldr \colon \mathbf{while} \; (\mathtt{TRUE}) \; \{ \\ & \quad \mathbf{either} \; Phase1a() \\ & \quad \mathbf{or} \quad \mathbf{with} \; (S \in \mathtt{SUBSET} \; Value) \; \{ \; Phase1c(S) \; \} \\ & \quad \mathbf{or} \quad \mathbf{with} \; (v \in Value) \; \{ \; Phase2a(v) \; \} \\ & \quad \} \\ & \quad \} \\ & \quad \} \\ & \quad \} \end{array}
```

4 Algorithm BPCon

In the abstract algorithm BPCon, we do not specify how acceptors learn what 1b messages have been sent. We simply introduce a variable knowsSent such that knowsSent[a] represents the set of 1b messages that (good) acceptor a knows have been sent, and have an action that nondeterministically adds sent 1b messages to this set.

--algorithm BPCon {

```
The variables:
      maxBal[a]
                    The highest ballot in which acceptor a has participated.
      maxVBal[a]
                    The highest ballot in which acceptor a has cast a vote (sent a 2b
                    message), or -1 if it hasn't cast a vote.
                    The value acceptor a has voted for in ballot maxVBal[a], or None
      maxVVal[a]
                    if maxVBal[a] = -1.
      2avSent[a]
                    A set of records in [val: Value, bal: Ballot] describing the 2av
                    messages that a has sent. A record is added to this set, and any
                     element with the same val field (and lower bal field) removed when
                     a sends a 2av message.
      knownSent[a] The set of 1b messages that acceptor a knows have been sent.
      bmsqs
                    The set of all messages that have been sent. See the discussion of
                     the msgs variable in module PConProof to understand our modeling
                     of message passing.
```

```
variables maxBal = [a \in Acceptor \mapsto -1],

maxVBal = [a \in Acceptor \mapsto -1],

maxVVal = [a \in Acceptor \mapsto None],

2avSent = [a \in Acceptor \mapsto \{\}],

knowsSent = [a \in Acceptor \mapsto \{\}],

bmsgs = \{\}

define \{

sentMsgs(type, bal) \triangleq \{m \in bmsgs : m.type = type \land m.bal = bal\}

KnowsSafeAt(ac, b, v) \triangleq
```

True for an acceptor ac, ballot b, and value v iff the set of 1b messages in knowsSent[ac] implies that value v is safe at ballot b in the Paxos consensus algorithm being emulated by the good acceptors. To understand the definition, see the definition of ShowsSafeAt in module PConProof and recall (a) the meaning of the mCBal and mCVal fields of a 1b message and (b) that the set of real acceptors in a ByzQuorum forms a quorum of the PCon algorithm.

```
LET S \triangleq \{m \in knowsSent[ac] : m.bal = b\}

IN \forall \exists BQ \in ByzQuorum :

\forall a \in BQ : \exists m \in S : \land m.acc = a

\land m.mbal = -1

\forall \exists c \in 0 .. (b-1) :

\land \exists BQ \in ByzQuorum :

\forall a \in BQ : \exists m \in S : \land m.acc = a

\land m.mbal \leq c

\land (m.mbal = c) \Rightarrow (m.mval = v)

\land \exists WQ \in WeakQuorum :

\forall a \in WQ :

\exists m \in S : \land m.acc = a

\land \exists r \in m.m2av : \land r.bal \geq c

\land r.val = v
```

We now describe the processes' actions as macros.

As in the Paxos consensus algorithm, a ballot self leader (good or malicious) can execute a Phasela action at any time.

```
macro Phase1a() { bmsgs := bmsgs \cup \{ [type \mapsto "1a", bal \mapsto self] \} \}
```

The acceptor's Phase1b ation is similar to that of the PCon algorithm.

```
\begin{aligned} & \textbf{macro } Phase1b(b) \text{ } \\ & \textbf{when } (b > maxBal[self]) \land (sentMsgs("1a",b) \neq \{\}) \text{ }; \\ & maxBal[self] := b \text{ }; \\ & bmsgs := bmsgs \ \cup \ \{ [type \mapsto "1b", \ bal \mapsto b, \ acc \mapsto self, \ m2av \mapsto 2avSent[self], \\ & mbal \mapsto maxVBal[self], \ mval \mapsto maxVVal[self]] \} \\ & \} \end{aligned}
```

A good ballot self leader can send a phase 1c message for value v if it knows that the messages in knowsSent[a] for a Quorum of (good) acceptors imply that they know that v is safe at ballot self, and that they can convince any other acceptor that the appropriate 1b messages have been sent to that it will also know that v is safe at ballot self.

A malicious ballot self leader can send any phase 1c messages it wants (including one that a good leader could send). We prove safety with a Phase1c action that allows a leader to be malicious. To prove liveness, we will have to assume a good leader that sends only correct 1c messages.

As in the PCon algorithm, we allow a Phase1c action to send a set of Phase1c messages. (This is not done in the Castro-Liskov algorithm, but seems natural in light of the PCon algorithm.)

```
\label{eq:macro-Phasel} \begin{array}{l} \mathbf{macro}\ Phaselc()\ \{\\ \mathbf{with}\ (S\in \mathtt{SUBSET}\ [type: \{ \text{``lc"}\},\ bal: self,\ val:\ Value])\ \{\\ bmsgs:=bmsgs\cup S\\ \}\\ \} \end{array}
```

If acceptor self receives a ballot b phase 1c message with value v, it relays v in a phase 2av message if

- it has not already sent a 2av message in this or a later ballot and
- ullet the messages in knowsSent[self] show it that v is safe at b in the non-Byzantine Paxos consensus algo- rithm being emulated.

```
\begin{aligned} \mathbf{macro} \; Phase 2av(b) \; \{ \\ \mathbf{when} \; \wedge \; maxBal[self] = < b \\ \; \wedge \; \forall \, r \in 2avSent[self] : r.bal < b \; ; \\ & \text{We could just as well have used } r.bal \neq b \; \text{in this condition.} \end{aligned}
\mathbf{with} \; \big( m \in \{ ms \in sentMsgs(\text{``1c''}, b) : KnowsSafeAt(self, b, ms.val) \} \; \big) \; \big\{ \\ \; bmsgs := bmsgs \; \cup \; \big\{ [type \mapsto \text{``2av''}, \; bal \mapsto b, \; val \mapsto m.val, \; acc \mapsto self] \, \big\}; \\ \; 2avSent[self] := \big\{ r \in 2avSent[self] : r.val \neq m.val \big\} \; \cup \; \big\{ [val \mapsto m.val, \; bal \mapsto b] \, \big\} \; \big\}; \\ \; maxBal[self] := b \\ \} \end{aligned}
```

Acceptor self can send a phase 2b message with value v if it has received phase 2av messages from a Byzantine quorum, which implies that a quorum of good acceptors assert that this is the first 1c message sent by the leader and that the leader was allowed to send that message. It sets maxBal[self], maxVBal[self], and maxVVal[self] as in the non-Byzantine algorithm.

```
\begin{aligned} \mathbf{macro} \; Phase2b(b) \; \{ \\ \mathbf{when} \; maxBal[self] = < b \; ; \\ \mathbf{with} \; & (v \in \{vv \in Value : \\ & \exists Q \in ByzQuorum : \\ & \forall aa \in Q : \\ & \exists m \in sentMsgs(\text{"2av"},b) : \land m.val = vv \\ & \land m.acc = aa\} \; ) \; \{ \\ bmsgs \; := bmsgs \; \cup \; \{ [type \mapsto \text{"2b"}, \; acc \mapsto self, \; bal \mapsto b, \; val \mapsto v] \} \; ; \\ maxVVal[self] \; := v \\ \} \; ; \\ maxBal[self] \; := b \; ; \end{aligned}
```

```
maxVBal[self] := b
```

}

At any time, an acceptor can learn that some set of 1b messages were sent (but only if they atually were sent).

```
macro LearnsSent(b) {
   with (S \in SUBSET \ sentMsgs("1b", b)) { knowsSent[self] := knowsSent[self] \cup S }
}
```

A malicious acceptor *self* can send any acceptor message indicating that it is from itself. Since a malicious acceptor could allow other malicious processes to forge its messages, this action could represent the sending of the message by any malicious process.

```
 \begin{array}{l} \mathbf{macro} \ FakingAcceptor() \ \{ \\ \mathbf{with} \ (m \in \{mm \in 1bMessage \cup 2avMessage \cup 2bMessage : mm.acc = self\}) \ \{ \\ bmsgs := bmsgs \cup \{m\} \\ \} \\ \} \\ \end{array}
```

We combine these individual actions into a complete algorithm in the usual way, with separate process declarations for the acceptor, leader, and fake acceptor processes.

```
process (acceptor ∈ Acceptor) {
    acc: while (TRUE) {
        with (b ∈ Ballot) {
            either Phase1b(b) or Phase2av(b) or Phase2b(b) or LearnsSent(b)
        }
      }
}

process (leader ∈ Ballot) {
      ldr: while (TRUE) { either Phase1a() or Phase1c() }
}

process (facceptor ∈ FakeAcceptor) {
      facc: while (TRUE) {FakingAcceptor() }
}
```

References

- [1] Leslie Lamport. Mechanically checked safety proof of a byzantine paxos algorithm. URL http://research.microsoft.com/users/lamport/tla/byzpaxos.html. The page can also be found by searching the Web for the 23-letter string obtained by removing the "-" from uid-lamportbyzpaxosproof.
- [2] Leslie Lamport, Dahlia Malkhi, and Lidong Zhou. Vertical paxos and primary-backup replication. In Srikanta Tirthapura and Lorenzo Alvisi, editors, *Proceedings of the 28th Annual ACM Symposium on Principles of Distributed Computing, PODC 2009*, pages 312–313. ACM, 2009.