SIMUW Theory of Equations: Symmetry and complex conjugates

Recall that $\overline{z}$ denotes the complex conjugate of the complex number z: if $z = x + iy$ with $x, y \in \mathbb{R}$, then $\overline{z} = x - iy$.

1. Find a polynomial with rational coefficients (not all zero) that has $\sqrt{2} + \sqrt{3}$ as a root.
2. What about $\sqrt{\sqrt{2} + (1 + \sqrt{5})/2}$? Or $\sqrt{2} + \sqrt{3} + \sqrt{5}$?
3. Prove that a complex number z is real if and only if $z = \overline{z}$.
4. Prove that if z and w are complex numbers, then $\overline{z + w} = \overline{z} + \overline{w}$ and $\overline{zw} = \overline{z} \cdot \overline{w}$.
5. What is the relationship between $|z|$ and $z\overline{z}$?
6. Prove that $(a^2 + b^2)(c^2 + d^2) = (ac - bd)^2 + (ad + bc)^2$. In other words, the product of two sums of two squares is always a sum of two squares. What does this have to do with complex numbers?
7. If $p(z)$ is a polynomial with real coefficients, then $\overline{p(z)} = p(\overline{z})$ for every complex number z.
8. If $p(z)$ is a polynomial with real coefficients, and if z_0 is a root of $p(z)$, then so is $\overline{z_0}$.