

SIMUW Theory of Equations: Straightedge and compass constructions

The rules of straightedge and compass constructions are as follows. You start with some set of given points (everything takes place in the plane). If P and Q are distinct points that are either given or previously constructed, you can construct the line PQ or the circle centered at P that goes through Q . You can construct the intersection of any two (distinct) lines or circles that have previously been constructed.

Note that you cannot use the compass to copy a segment! In particular, you cannot set the compass to a certain distance, pick it up, and then use it to draw a circle of that radius somewhere else. The compass collapses whenever you pick it up. (This is a technicality, but a technicality worth thinking about. This restriction is traditional. Problem 3 below shows that it is not really restrictive, since one can get around it.)

Being given a segment means being given its endpoints, and being given a circle means being given its center and a point on it.

1. Given distinct points P and Q , construct the perpendicular bisector of the segment PQ .
2. Given distinct points P and Q and a point R not on the line PQ , construct the line through R parallel to PQ .
3. Given points P, Q, R, S with $P \neq Q$ and $R \neq S$, construct the point on the ray RS whose distance from R equals the distance from P to Q .
4. Given noncollinear points P, Q, R , construct the bisector of the angle PQR centered at Q .
5. Given segments of length x and y , construct a segment of length $x + y$.
6. Given segments of length x and y with $x > y$, construct a segment of length $x - y$.
7. Given segments of length 1, x , and y , construct a segment of length xy . (Hint: hfr fvzvyne gevnatyrf.)
8. Why is problem 7 impossible just given segments of length x and y ?
9. Given segments of length 1, x , and y , construct a segment of length x/y .
10. Given segments of length 1 and x , construct a segment of length \sqrt{x} . (Hint: guvax nobhg gur trbzrgevp cebbs bs gur vardhnyvgl sebz Ghrfqnl.)

Optional: given three circles in the plane, construct each circle that is tangent to all three. (There are generally eight such circles, since they can be internally or externally tangent to each of the three original circles. If you go about this the wrong way it's extremely difficult, but if you do it right it's manageable.)