

SIMUW Theory of Equations: Coefficients of equations

Suppose x_1, x_2, x_3, x_4 are the roots of $x^4 + ax^3 + bx^2 + cx + d = 0$. For each of the following quantities, compute it in terms of a, b, c, d or explain why that cannot be done. For example, $x_1 + x_2 + x_3 + x_4 = -a$ and $x_1x_2x_3x_4 = d$.

1. $x_1^2 + x_2^2 + x_3^2 + x_4^2$
2. $x_1^3 + x_2^3 + x_3^3 + x_4^3$
3. $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4}$
4. $x_1x_2 + x_2x_3 + x_3x_4 + x_4x_1$
5. $(x_1 + x_2)(x_1 + x_3)(x_1 + x_4)(x_2 + x_3)(x_2 + x_4)(x_3 + x_4)$
6. $(x_1x_2 + x_3x_4)(x_1x_3 + x_2x_4)(x_1x_4 + x_2x_3)$