

SIMUW Theory of Equations: Cubic and quartic equations

1. How many of the roots of $x^3 + 3x - 2 = 0$ are real numbers? Find formulas for them (in terms of radicals).
2. Use Cardano's method to solve the cubic $x^3 + x - 2 = 0$. Explain how your answer agrees with the obvious fact that $x = 1$ is a solution.

3. Simplify

$$\sqrt[3]{\sqrt{243} + \sqrt{242}} - \sqrt[3]{\sqrt{243} - \sqrt{242}}$$

as much as possible.

4. (optional) If you want to test your manipulation abilities some more, solve $x^3 + x^2 + x + 1$ using Cardano's formula and compare with the roots -1 , i , and $-i$.
5. Solve the quartic equation $x^4 + 4x - 1 = 0$ using Ferrari's method. The resolvent cubic should have a rational root (so you won't need to apply Cardano's formula).
6. Solve $x^5 + x^4 - 2x^3 - 2x^2 + 1 = 0$.