On the economic payoff of forensic systems when used to trace counterfeited software and content

Yacov Yacobi
MSR
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Abstract

We analyze how well forensic systems reduce counterfeiting (CF) of SW and content. We make a few idealized assumptions, and show that if the revenues of the producer before the introduction of forensics \( R_o \) are non-zero then the payoff of forensics is independent of the overall market size, it declines as the ratio between the penalty and the crime (both monetized) goes up, but that this behavior is reversed if \( R_o = 0 \). We also show that the payoff goes up as the ratio between success probability with and without forensics grows, however, for typical parameters most of the payoff is already reached when this ratio is 5.

1 Introduction

We analyze how well forensic systems reduce counterfeiting (CF) of SW and content. Unlike the pirate, the CF pretends to be the legitimate producer, charges the same price as the producer, and in general competes with the producer in the same market (that does not include the market share of the pirate). We make the following assumptions: (i) audit events are independent of each other, (ii) False positives are negligible (at the expense of higher false negatives); (iii) Once caught and successfully prosecuted, the magnitude of the whole theft of an independent CF group is known and the penalty is in some fixed proportion, \( \gamma \), to the theft.

Enforcement mechanisms, such as screening and registration, as well as raising the initial bar (e.g. by adding sophisticated holograms) push down the number of counterfeiters, \( n \), however, forensics have no effect on \( n \) if the CF are rational, since as is shown later, CF has positive expected profit for any value of \( q_w \) and \( \gamma \).

We show that if the revenues of the producer before the introduction of forensics \( (R_o) \) are non-zero then the payoff of forensics is

\[
\lambda n p \left[ \frac{1}{\ln(1-q_o)} - \frac{1}{\ln(1-q_w)} \right],
\]

where \( \lambda \) is a constant that depends only on \( \gamma \), \( n \) = the number of independent CF groups, \( p \) = the selling price of one copy of the protected object, \( q_w, q_o \) are the probability of audit, detection, and successful prosecution of a single illegal
copy, with and without forensics, respectively. The payoff is independent of the overall market size, it declines as $\gamma$ goes up (this behavior is reversed if $R_0 = 0$), and goes up as $k = q_{\text{in}}/q_0$ grows, however, for typical parameters most of the potential payoff is already reached when $k = 5$.

In section 2 we develop the payoff function of the counterfeiter, find its optimum, and show that it is always positive. In section 3 we develop the payoff function of the forensic system. This work was heavily influenced by unpublished manuscripts [HV1,HV2].

2 The economics of the counterfeiter

Let $x = \text{the number of copies made by the counterfeiter}$; $F = \text{the penalty when caught}$. For each copy let the probability to audit an illegal copy, make a correct decision, and successfully prosecute be $q < 1$.

Let $\pi(x)$ be the probability to audit an illegal copy, make a correct decision, and successfully prosecute after the counterfeiter has sold $x$ illegal copies.

When the audit events are independent (as we henceforth assume) $\pi(x) = (1 - q)^x$. Let $p = \text{the price of a single copy (legal and illegal)}$; Assuming zero distribution costs for the counterfeiter, the gain function of the counterfeiter is [HV1]

$$P(x) = (1 - \pi(x))px - \pi(x)F,$$ (1)

If there are dependencies between audit events this function becomes an upper bound on the gain function. Let $x^*$ denote the value of $x$ for which $P(x)$ reaches its maximum. In reality $x$ has only integer values, we first pretend that $x$ can have any real value, so that we can use differentials to approximate the behavior of $P(x)$. At the end we get back to the case of integer $x$ [KJ].

Let $L(x)$ be the Lambert function, i.e., that function for which

$$L(e^{x/1+\gamma}) = x.$$

Theorem 1 $x^* = \min\{D(p), \frac{\lambda}{\ln(1-q)}\}$.

Proof. Clearly $x^* \leq D(p)$. $P(x) = [1-\pi(x)(1+\gamma)]px = [(1-q)^x(1+\gamma)-\gamma]px$.

$P'(x) = (1-q)^x(1+\gamma)p[x\ln(1-q)+1]-\gamma p$. Let $x^* = \frac{L(e^{\gamma/1+\gamma})}{\ln(1-q)}$. Then $P'(x^*) = 0$. $P''(x^*) = (1-q)^x p(x^* \ln(1-q) + 2)(1+\gamma) \ln(1-q) < 0 \text{ (since } \ln(1-q) < 0 \text{ and } x^* \ln(1-q) + 2 = \frac{L(e^{\gamma/1+\gamma})}{1+\gamma} + 1 > 0)$, implying that $P(x^*)$ is a maximum.

Note that $\lambda < 0$ and hence $x^* > 0$. There is no value of real $0 < \gamma$ for which $L(e^{\gamma/1+\gamma}) = 1$, although $\lim_{\gamma \to \infty} L(e^{\gamma/1+\gamma}) = L(e) = 1$. In other words, when the penalty goes to infinity the counterfeiter’s gain goes to zero as expected.

Theorem 2 $0 < P(x^*)$. 

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Proof. \( P(0) = 0, P'(0) = p > 0 \) and \( 0 < x^* \).

The above is true for the continuous function approximating the realistic \( P(x) \), but if \( q(1 + \gamma) > 1 \) then \( 0 < x^* < 1 \), and for all integer \( x \), \( P(x) \leq 0 \) [KJ]. Note that in reality \( q \ll 1 \), and \( \gamma \) is small (e.g. \( \gamma = 3 \) in triple damage), so \( q(1 + \gamma) > 1 \) doesn’t happen.

When the counterfeiter has costs \( c > 0 \) it shifts the \( P(x) \) curve down by \( c \) without affecting \( x^* \).

3 The economics of the forensic system

We initially analyze a toy problem involving only one counterfeiter, and later generalize to multiple counterfeiters.

3.1 Notations

The average selling price is \( p \). We use subscripts \( w, o \) to denote parameter values with and without forensic system, respectively. Let \( x \) be the market size of the counterfeiter and \( D \) denote the market size shared by the counterfeiter and the producer (where the producer has \( D \) of the market). For \( i \in \{w, o\} \), \( \pi_i(x) = Pr[\text{detect} \& \text{penalize counterfeaters who resold} \ x \ \text{copies}] \); \( F_i = \text{penalty} \)

Let \( R_i \), denote the revenues of the producer, then the payoff of the producer due to the introduction of forensics is

\[ P_2 = R_w - R_o. \]

3.2 A Single counterfeiter

Assuming the counterfeiter maximizes her gain, \( R_w = (D - x^*_w)p \), and \( R_o = (D - x^*_o)p \), hence \( R_w - R_o = p(x^*_w - x^*_o) \), and in those cases where \( D_i \geq \frac{\lambda}{\ln(1 - q_i)} \)

\[ P_2 = \lambda p \left[ \frac{1}{\ln(1 - q_o)} - \frac{1}{\ln(1 - q_w)} \right] \]

independent of \( D \). Otherwise \( D_i \) is substituted for \( x^*_i \).

We look at \( P_2 = R_w - R_o \) in three cases depending on the value of \( D \) relative to the interval \( [x^*_w, x^*_o] \) (it is always the case that \( x^*_w \leq x^*_o \)).

1. If \( D < x^*_w \) (regardless of \( D_o \) and of \( x^*_o \) so we cover here two cases): then the CF owns the whole market even with forensics, and \( P_2 = 0 \).

2. If \( x^*_w \leq D < x^*_o \) then \( P_2 = (D_o - \frac{\lambda}{\ln(1 - q_w)})p \), which grows with \( \gamma \). Here the situation is hopeless without forensics, and once we have some forensics the harsher the penalties the higher the gain from forensics.

3. If \( x^*_w \leq x^*_o \leq D \) then \( P_2 = \lambda p \left[ \frac{1}{\ln(1 - q_o)} - \frac{1}{\ln(1 - q_w)} \right] \), which goes down as \( \gamma \) grows. It is a situation where even without forensics we have significant probability (\( q_o \)) to detect and punish fraud.
For any fixed value of $\gamma$ and $q_o = 10^{-6}$ the payoff $p = \lambda p[\frac{1}{\ln(1-q_w)} - \frac{1}{\ln(1-kq_o)}]$ captures most of its potential value when $k = q_w/q_o = 5$ (see Fig. 1 below), and further increase in $k$ (which can mean a significant increase in cost) adds little additional payoff.

Fig. 1: Payoff as a function of $k$, for $q_o = 10^{-6}$; $p = 10$; $\gamma = 1$.

### 3.3 Many counterfeiters

Each CF group sells the optimum $x_i^* = \frac{\lambda}{\ln(1-q_i)}$ copies, as before, independent of $n$, so $n$ CF groups sell $nx^*$. nobody needs more than one copy, and the detection probability for each CF group is $\pi(x_i^*)$. In the three cases above in the condition clauses we have to substitute $D/n$ for $D$, and in the payoff expression we need to substitute $nx_i^*$ for $x_i^*$. The result is:

**Theorem 3** For $n$ CFs: (i) If $D/n < x_w^*$ then $P_2 = 0$, (ii) If $x_w^* \leq D/n < x_o^*$ then $P_2 = (D_o - n\lambda \ln(1-q_w)/\ln(1-q_o))p$, which grows with $\gamma$, (iii) If $x_w^* \leq x_o^* \leq D/n$ then $P_2 = \lambda np[\frac{1}{\ln(1-q_o)} - \frac{1}{\ln(1-kq_o)}]$, which goes down as $\gamma$ grows.

Fig 2: Payoff as a function of $\gamma$. For $q_o = 10^{-6}$, $p = 10$, $k = 5$, $n = 50$.

Too many counterfeiters can crowd out the producer and even each other. The average market in which a CF competes with the producer is of size $D/n$. When $D_i < \frac{n\lambda}{\ln(1-q_i)}$ for $i = o,w$ the producer is crowded out. And when $P(D/n) = (1 - \pi(D/n))pD/n - \pi(D/n)F = 0$ the CF crowd out each other and their returns disappear. This happens when $n_{max} = \frac{D\ln(1-q)}{\ln(1-kq_o)}$. This can help estimate the legal expenses for prosecuting all the potential CF groups.
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4 References


