Unabridged version of Table 1.

Table 1: Quantitative analysis of reconstruction quality. Kernel properties are degree N, width W, and approximation order L. The main columns report the mean structural similarity (MSSIM) between five reference images (circles of Figure 3, four Kodak benchmarks) and their reconstructions for three types of experiments (repeated translations or rotations, and single upsampling). The kernels are sorted in descending order of average quality across all experiments. The interpolating B-splines `bspline*i` consistently outperform the more traditional filters for the same N and W. The O-MOMS kernels `omoms*` offer even slightly higher quality but at the expense of differentiability. The quasi-interpolant `condat2`, which has degree 2 and support 3, also performs remarkably well.

				Comparison against ground truth (MSSIM)											
	Properties			Repeated translations					Repeated rotations					Upscaling	Average
Kernel	N	W	L	CIR	K05	K08	K19	K23	CIR	K05	K08	K19	K23	CIR	AVG
`omoms5`	5	6	6	0.993	0.975	0.945	0.957	0.984	0.999	0.986	0.977	0.980	0.990	0.886	0.970
`somoms5`	5	6	6	0.991	0.968	0.933	0.948	0.981	0.998	0.984	0.973	0.977	0.989	0.886	0.966
`bspline5i`	5	6	6	0.990	0.966	0.931	0.947	0.981	0.998	0.983	0.972	0.976	0.989	0.886	0.965
`omoms4`	4	5	5	0.989	0.963	0.925	0.942	0.980	0.997	0.981	0.969	0.974	0.988	0.886	0.963
`somoms4`	4	5	5	0.988	0.961	0.922	0.941	0.979	0.997	0.981	0.969	0.974	0.988	0.886	0.962
`omoms3`	3	4	4	0.981	0.949	0.905	0.929	0.975	0.997	0.980	0.968	0.973	0.987	0.886	0.957
`bspline4i`	4	5	5	0.982	0.950	0.906	0.929	0.976	0.995	0.977	0.963	0.969	0.986	0.886	0.956
`quasiblu35`	3	4	4	0.965	0.926	0.874	0.907	0.969	0.994	0.975	0.961	0.968	0.986	0.885	0.946
`condat3`	3	4	4	0.962	0.921	0.867	0.903	0.967	0.991	0.971	0.956	0.964	0.984	0.885	0.943
`schaum6`	6	7	7	0.958	0.915	0.859	0.898	0.966	0.985	0.964	0.946	0.957	0.982	0.885	0.938
`hamming6`	–	6	1	0.960	0.918	0.862	0.901	0.966	0.977	0.956	0.937	0.951	0.979	0.885	0.936
`schaum7`	7	8	8	0.958	0.915	0.859	0.898	0.966	0.976	0.956	0.936	0.950	0.979	0.885	0.934
`bspline3i`	3	4	4	0.948	0.904	0.846	0.888	0.963	0.977	0.958	0.938	0.952	0.980	0.885	0.931
`condat2`	2	3	3	0.930	0.884	0.822	0.872	0.957	0.989	0.967	0.951	0.961	0.983	0.884	0.927
`schaum4`	4	5	5	0.921	0.875	0.809	0.864	0.955	0.967	0.949	0.927	0.944	0.977	0.884	0.916
`schaum5`	5	6	6	0.921	0.875	0.809	0.864	0.955	0.950	0.936	0.912	0.934	0.973	0.885	0.910
`lanczos6`	–	6	1	0.964	0.788	0.787	0.827	0.909	0.987	0.959	0.946	0.958	0.978	0.884	0.908
`keys6`	3	6	4	0.909	0.863	0.795	0.854	0.952	0.951	0.937	0.912	0.934	0.974	0.885	0.906
`bspline2i`	2	3	3	0.901	0.854	0.787	0.848	0.949	0.955	0.939	0.916	0.936	0.975	0.884	0.904
`hann6`	–	6	1	0.925	0.778	0.779	0.825	0.910	0.967	0.946	0.932	0.947	0.974	0.883	0.897
`omoms2`	2	3	3	0.814	0.774	0.713	0.793	0.929	0.961	0.944	0.921	0.940	0.976	0.885	0.877
`meijering7`	7	8	3	0.850	0.806	0.735	0.811	0.937	0.917	0.915	0.886	0.916	0.967	0.883	0.875
`blackman6`	–	6	1	0.842	0.800	0.728	0.806	0.935	0.914	0.913	0.883	0.914	0.967	0.883	0.871
`meijering5`	5	6	3	0.832	0.792	0.719	0.799	0.933	0.905	0.907	0.877	0.910	0.965	0.883	0.866
`schaum2`	2	3	3	0.822	0.782	0.710	0.792	0.930	0.921	0.914	0.885	0.916	0.967	0.877	0.865
`welch4`	–	4	1	0.896	0.683	0.708	0.761	0.865	0.945	0.922	0.909	0.931	0.964	0.878	0.860
`lanczos4`	–	4	1	0.822	0.782	0.710	0.792	0.930	0.896	0.902	0.871	0.906	0.963	0.882	0.860
`keys`	3	4	3	0.822	0.782	0.710	0.792	0.930	0.894	0.900	0.869	0.905	0.963	0.882	0.859
`schaum3`	3	4	4	0.822	0.782	0.710	0.792	0.930	0.876	0.891	0.858	0.897	0.960	0.883	0.855
`dalai1`	1	2	2	0.657	0.652	0.601	0.715	0.896	0.956	0.938	0.915	0.936	0.974	0.865	0.828
`linrev`	1	2	2	0.686	0.667	0.587	0.710	0.898	0.960	0.926	0.903	0.924	0.960	0.864	0.826
`condat1`	1	2	2	0.651	0.648	0.597	0.713	0.895	0.947	0.933	0.909	0.931	0.972	0.866	0.824
`welch6`	–	6	1	0.948	0.498	0.532	0.547	0.668	0.985	0.941	0.930	0.949	0.970	0.884	0.805
`hamming4`	–	4	1	0.663	0.657	0.603	0.716	0.897	0.822	0.859	0.826	0.875	0.951	0.879	0.795
`blackmanharris6`	–	6	1	0.657	0.652	0.599	0.714	0.896	0.808	0.852	0.819	0.870	0.949	0.879	0.790
`hann4`	–	4	1	0.550	0.579	0.536	0.674	0.876	0.730	0.812	0.780	0.843	0.936	0.875	0.744
`mitchell`	3	4	2	0.581	0.599	0.554	0.685	0.881	0.625	0.761	0.733	0.810	0.921	0.881	0.730
`blackman4`	–	4	1	0.458	0.520	0.485	0.643	0.859	0.666	0.780	0.751	0.822	0.927	0.860	0.706
`idodgson`	2	3	2	0.391	0.480	0.449	0.623	0.847	0.693	0.793	0.763	0.831	0.931	0.866	0.697
`blackmanharris4`	–	4	1	0.413	0.493	0.461	0.629	0.851	0.634	0.765	0.737	0.813	0.921	0.840	0.687
`linear`	1	2	2	0.391	0.480	0.449	0.623	0.847	0.540	0.721	0.698	0.787	0.908	0.864	0.664
`adodgson`	2	3	?	0.391	0.480	0.449	0.623	0.847	0.448	0.678	0.659	0.764	0.894	0.872	0.646
`bartlett6`	–	6	1	0.279	0.428	0.447	0.622	0.831	0.475	0.698	0.684	0.782	0.899	0.874	0.638
`gaussian2p5`	–	2.5	1	0.385	0.476	0.445	0.621	0.846	0.526	0.715	0.692	0.430	0.906	0.856	0.627
`condat0`	0	1	1	0.042	0.146	0.130	0.433	0.607	0.535	0.712	0.689	0.781	0.900	0.593	0.506
`nearest`	0	1	1	0.042	0.102	0.087	0.367	0.560	0.547	0.654	0.633	0.731	0.851	0.586	0.469
`bartlett4`	–	4	1	0.735	0.069	0.061	0.042	0.022	0.828	0.470	0.497	0.457	0.437	0.852	0.406

PSNR results for the same experiments as in Table 1 (unabridged).

Quantitative analysis of reconstruction quality using PSNR. This table presents similar results to Table 1, but using the PSNR metric (based on mean squared error) rather than the perceptual MSSIM metric.

				Comparison against ground truth (PSNR)											
	Properties			Repeated translations					Repeated rotations					Upscaling	Average
Kernel	N	W	L	CIR	K05	K08	K19	K23	CIR	K05	K08	K19	K23	CIR	AVG
omoms5	5	6	6	34.21	34.02	29.87	34.65	40.53	41.08	35.86	33.31	38.34	40.45	21.31	34.88
somoms5	5	6	6	32.15	33.00	29.04	33.73	39.86	37.71	35.06	32.58	37.50	39.76	21.31	33.79
bspline5i	5	6	6	31.81	32.82	28.90	33.58	39.74	37.16	34.91	32.45	37.34	39.64	21.31	33.60
omoms4	4	5	5	30.98	32.40	28.55	33.21	39.44	35.61	34.48	32.06	36.89	39.27	21.31	33.11
somoms4	4	5	5	30.59	32.21	28.41	33.06	39.31	35.31	34.39	31.97	36.80	39.19	21.31	32.96
omoms3	3	4	4	28.55	31.13	27.53	32.16	38.51	35.43	34.25	31.84	36.63	39.10	21.31	32.40
bspline4i	4	5	5	28.60	31.19	27.58	32.22	38.55	32.82	33.59	31.23	35.97	38.51	21.31	31.96
quasiblu35	3	4	4	25.62	29.61	26.32	30.97	37.27	32.45	33.37	31.02	35.74	38.32	21.31	31.09
condat3	3	4	4	25.16	29.34	26.09	30.75	37.04	30.52	32.74	30.45	35.10	37.79	21.31	30.57
schaum6	6	7	7	24.83	29.07	25.80	30.49	36.79	28.47	31.85	29.61	34.19	37.02	21.30	29.95
hamming6	–	6	1	25.14	29.23	25.88	30.58	36.91	26.88	31.18	28.98	33.52	36.41	21.30	29.64
condat2	2	3	3	22.51	27.82	24.83	29.47	35.63	29.79	32.24	29.95	34.57	37.36	21.29	29.59
schaum7	7	8	8	24.83	29.07	25.80	30.49	36.79	26.77	31.09	28.89	33.43	36.38	21.30	29.53
bspline3i	3	4	4	23.77	28.56	25.46	30.11	36.33	26.80	31.25	29.03	33.59	36.49	21.31	29.34
schaum4	4	5	5	22.07	27.50	24.51	29.18	35.33	25.39	30.49	28.29	32.83	35.85	21.28	28.43
schaum5	5	6	6	22.07	27.50	24.51	29.18	35.33	23.73	29.66	27.49	32.02	35.15	21.29	27.99
keys6	3	6	4	21.44	27.14	24.21	28.85	34.96	23.75	29.69	27.51	32.04	35.17	21.29	27.82
bspline2i	2	3	3	21.00	26.91	24.07	28.66	34.72	24.00	29.85	27.67	32.19	35.29	21.29	27.79
lanczos6	–	6	1	22.20	22.64	22.20	26.02	31.05	26.72	31.19	29.03	33.33	37.07	21.29	27.52
omoms2	2	3	3	18.21	25.25	22.80	27.22	32.82	24.58	30.16	27.98	32.51	35.56	21.30	27.13
meijering7	7	8	3	19.22	25.81	23.12	27.59	33.54	21.54	28.55	26.39	30.86	34.14	21.27	26.55
blackman6	–	6	1	19.01	25.69	23.02	27.47	33.39	21.37	28.46	26.30	30.77	34.05	21.27	26.44
schaum2	2	3	3	18.50	25.36	22.73	27.13	33.02	21.65	28.49	26.33	30.77	34.05	21.18	26.29
hann6	–	6	1	19.74	22.35	21.92	25.78	30.10	23.54	29.64	27.68	31.68	35.40	21.27	26.28
meijering5	5	6	3	18.77	25.53	22.88	27.30	33.21	20.97	28.23	26.07	30.53	33.84	21.26	26.24
lanczos4	–	4	1	18.50	25.36	22.73	27.13	33.02	20.59	28.02	25.88	30.32	33.64	21.26	26.04
dalai1	1	2	2	15.70	23.53	21.31	25.60	30.81	23.98	29.62	27.49	31.90	35.12	21.17	26.02
keys	3	4	3	18.50	25.36	22.73	27.13	33.02	20.52	27.96	25.81	30.25	33.59	21.25	26.01
schaum3	3	4	4	18.50	25.36	22.73	27.13	33.02	19.89	27.61	25.47	29.88	33.26	21.26	25.83
condat1	1	2	2	15.64	23.48	21.27	25.55	30.75	23.19	29.35	27.22	31.63	34.86	21.17	25.83
linrev	1	2	2	15.79	23.59	21.00	25.18	30.86	22.02	28.29	26.25	30.32	33.91	21.02	25.29
welch6	–	6	1	20.95	16.31	16.68	19.98	24.86	26.27	29.65	27.59	32.08	35.65	21.31	24.67
hamming4	–	4	1	15.79	23.59	21.34	25.61	30.88	18.28	26.71	24.62	28.88	32.28	21.22	24.47
welch4	–	4	1	18.36	19.92	20.01	23.78	27.69	21.27	27.73	25.92	29.78	33.50	21.23	24.47
blackmanharris6	–	6	1	15.71	23.54	21.30	25.57	30.82	17.95	26.50	24.43	28.67	32.06	21.22	24.34
hann4	–	4	1	14.61	22.71	20.58	24.86	29.85	16.56	25.59	23.57	27.71	31.09	21.18	23.48
mitchell	3	4	2	14.90	22.93	20.78	25.05	30.11	15.23	24.62	22.68	26.74	30.04	21.14	23.11
blackman4	–	4	1	13.86	22.09	20.03	24.36	29.16	15.69	24.96	22.99	27.08	30.40	21.05	22.88
idodgson	2	3	2	13.39	21.67	19.65	24.02	28.70	16.03	25.22	23.23	27.34	30.69	21.12	22.82
blackmanharris4	–	4	1	13.54	21.81	19.78	24.13	28.84	15.32	24.67	22.73	26.80	30.08	20.91	22.60
linear	1	2	2	13.39	21.67	19.65	24.02	28.70	14.40	23.94	22.06	26.12	29.31	20.97	22.20
adodgson	2	3	?	13.39	21.67	19.65	24.02	28.70	13.65	23.24	21.43	25.50	28.56	20.72	21.87
gaussian2p5	–	2.5	1	13.35	21.64	19.62	23.98	28.65	14.28	23.84	21.97	22.77	29.20	20.88	21.84
bartlett6	–	6	1	12.69	20.91	19.26	23.71	27.82	13.86	23.40	21.63	25.71	28.66	21.26	21.72
condat0	0	1	1	10.01	14.51	12.57	17.94	17.21	14.22	23.66	21.82	25.87	28.90	19.28	18.73
nearest	0	1	1	8.37	13.68	11.75	16.98	16.84	11.78	20.80	19.01	23.10	25.88	19.17	17.03
bartlett4	–	4	1	12.99	6.47	6.59	6.82	6.49	14.39	10.24	10.42	11.71	13.90	20.91	10.99

Results of transient and frequency response for the extended list of filter kernels

We partition the kernels into 3 sections:

Interpolating schemes All these kernels are piecewise polynomial. Each kernel φ satisfies $\varphi(0) = 1$ and $\varphi(k) = 0$ for all $k \in \mathbb{Z} \setminus \{0\}$.

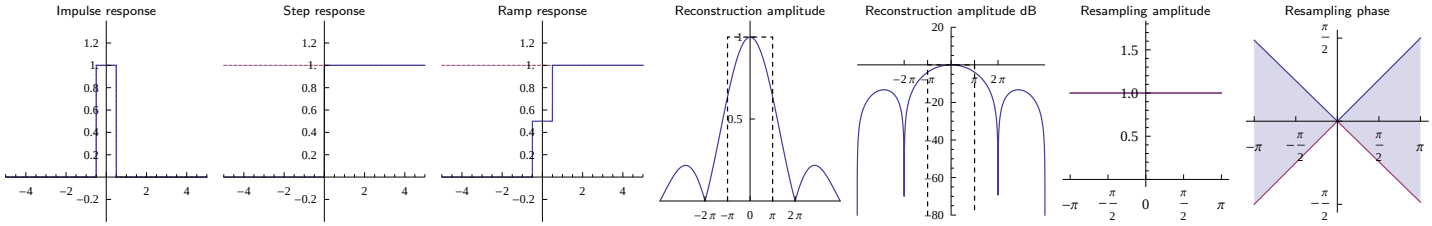
Approximating schemes These kernels are also piecewise polynomial, but not interpolating. We also included the Gaussian kernel in this set.

Windowed sinc These are all sinc functions modulated by a variety of different windows.

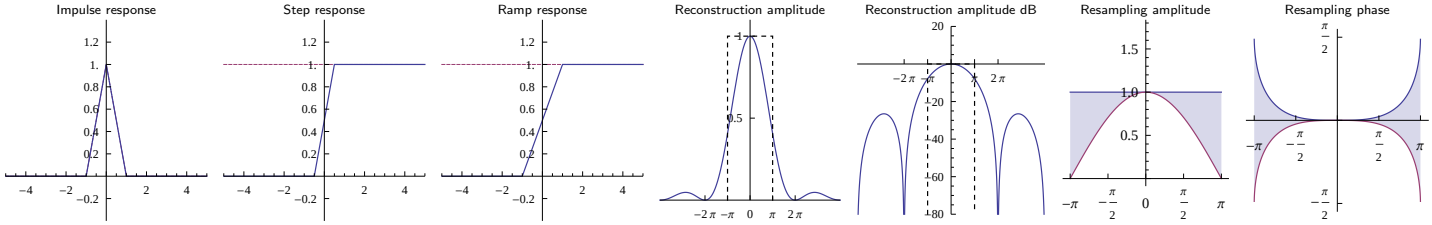
The red curves, if present, indicate the simpler equivalent basis functions φ when the kernel $\overline{\varphi}$ can be factored with a discrete filter as $\overline{\varphi} = \boldsymbol{q} * \varphi$.

Interpolating schemes

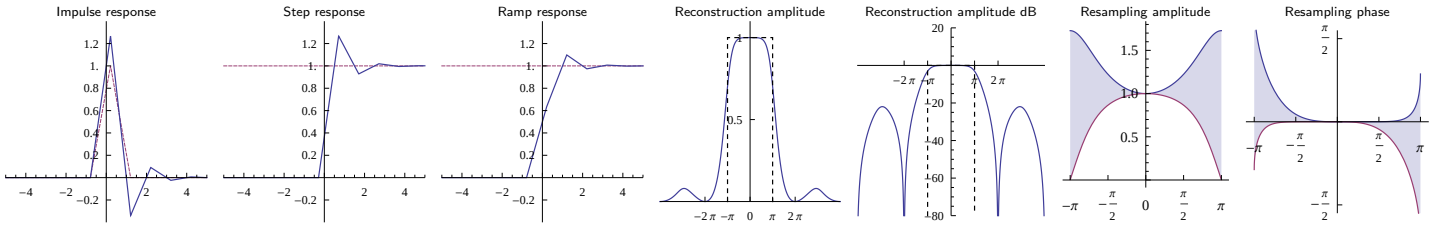
`nearest`: Nearest neighbor (also B-spline $\beta^0 = (\beta^0)_{\text{int}}$)



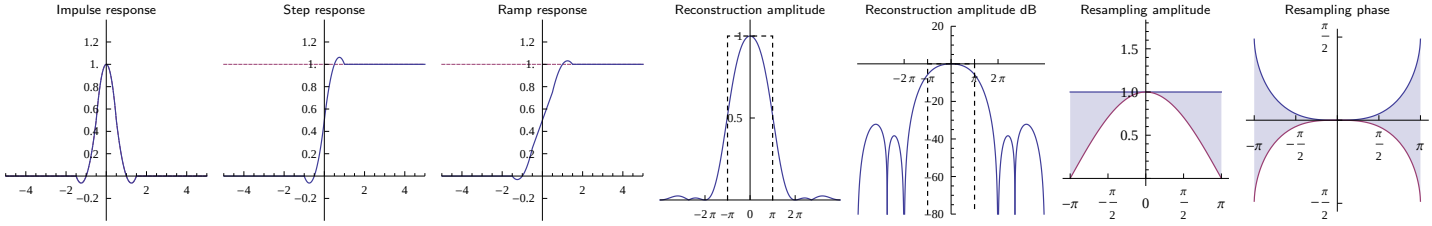
`linear`: Linear interpolation (also B-spline $\beta^1 = (\beta^1)_{\text{int}}$)



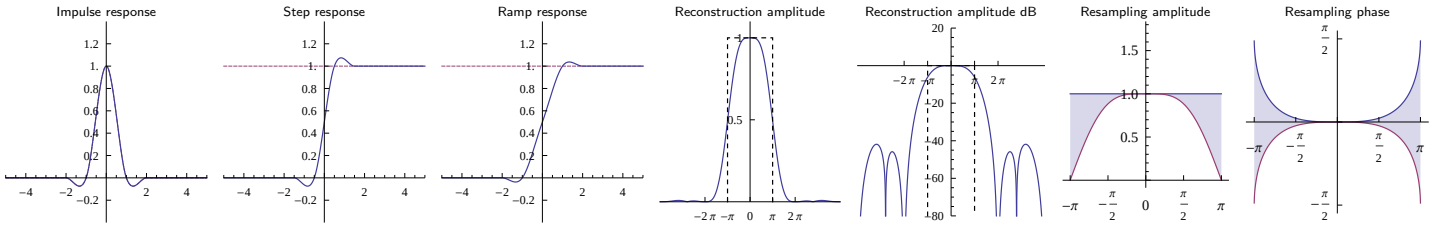
`linrev`: Linear interpolation revitalized [Blu et al. 2004]



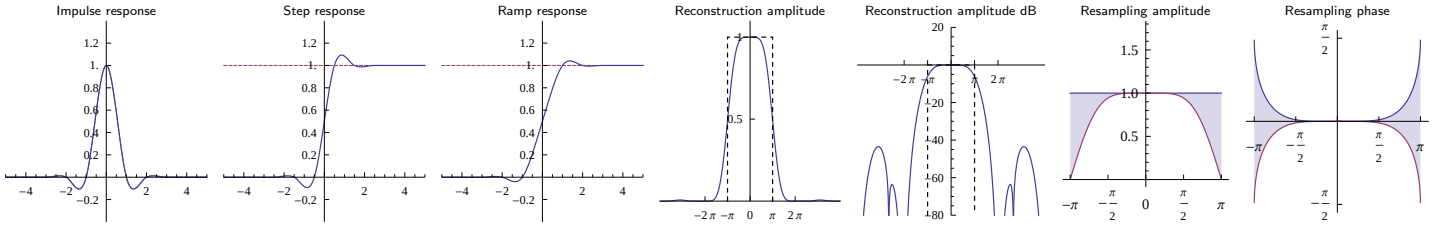
`idodgson`: Interpolating quadratic of Dodgson [1997]



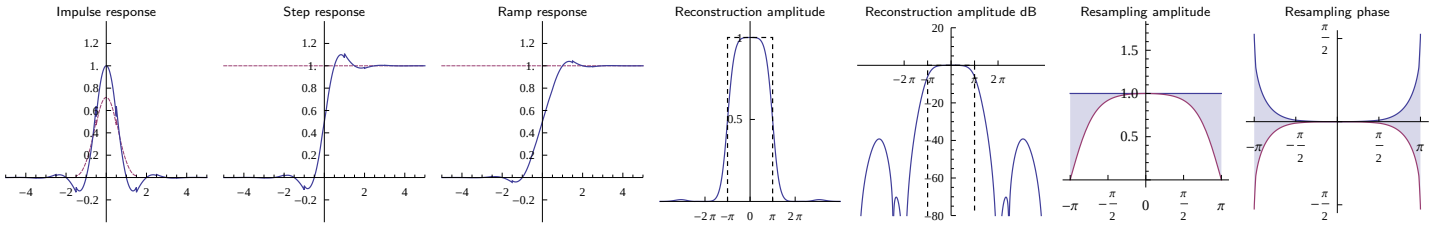
`keys`: Keys [1981] (also Catmull-Rom [Catmull and Rom 1974])



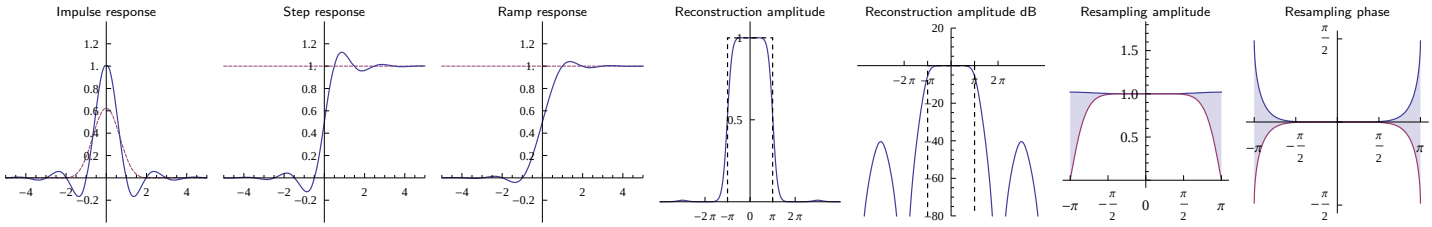
`keys6`: Keys [1981] with $W = 6, L = 4$



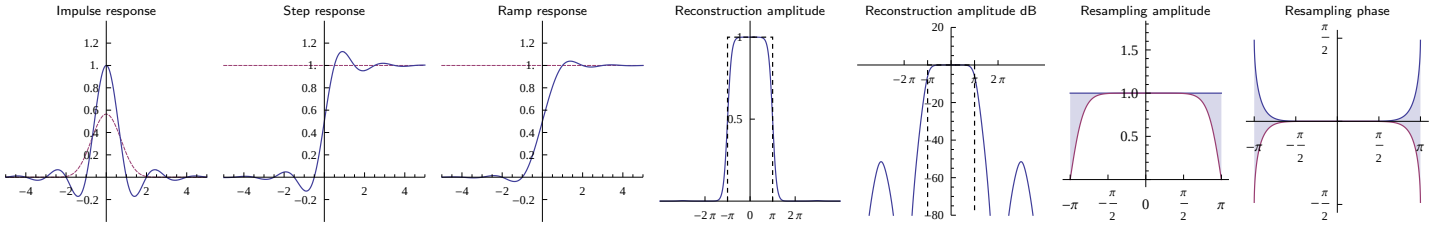
`omoms2`: O-MOMS-2 of [Blu et al. 2001]



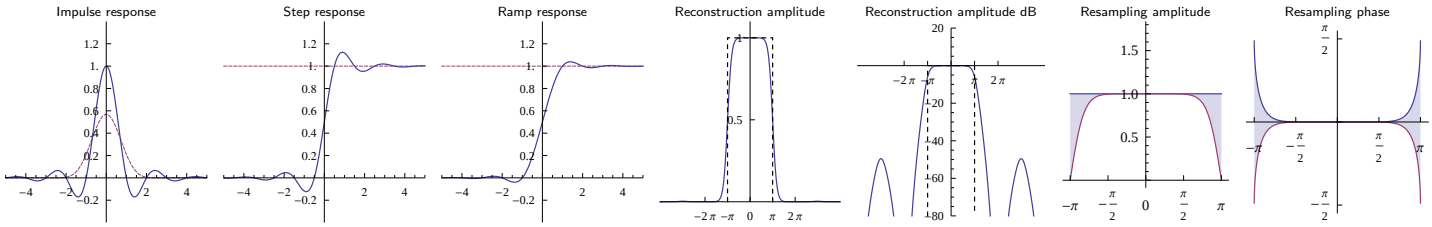
`omoms3`: O-MOMS-3 of [Blu et al. 2001]



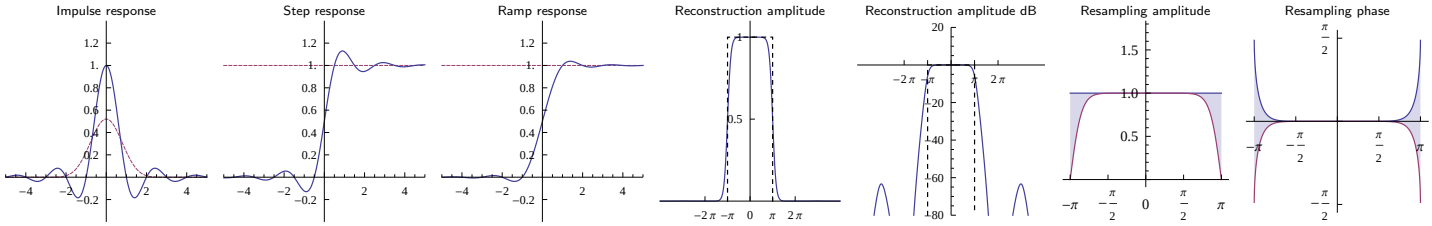
`omoms4`: O-MOMS-4 of [Blu et al. 2001]



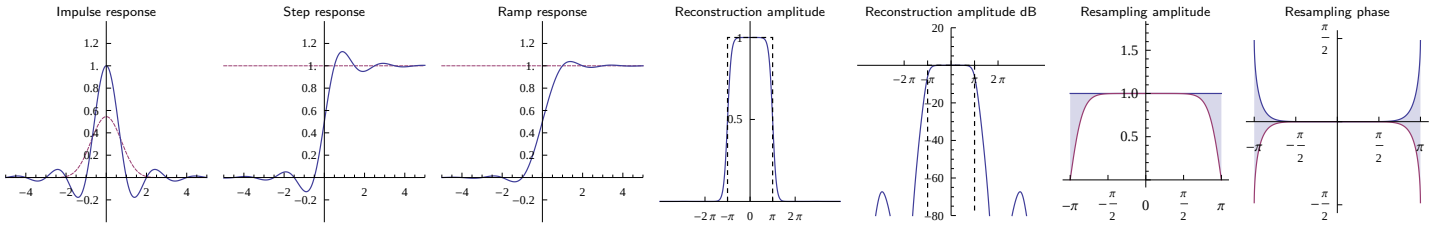
`somoms4`: SO-MOMS-4 of [Blu et al. 2001]



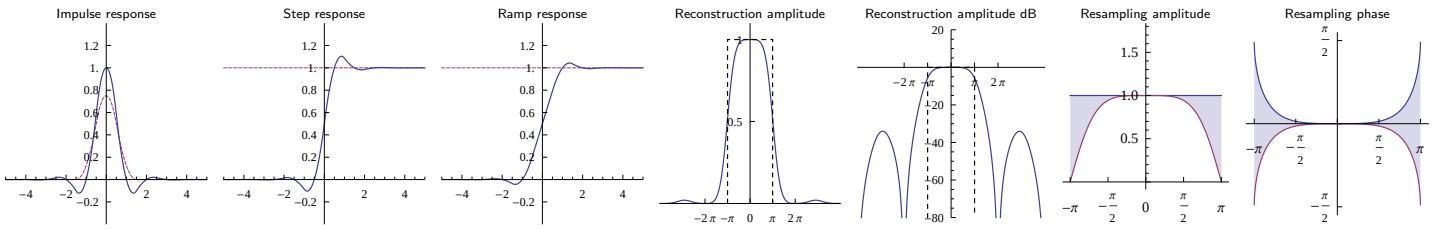
`omoms5`: O-MOMS-5 of [Blu et al. 2001]



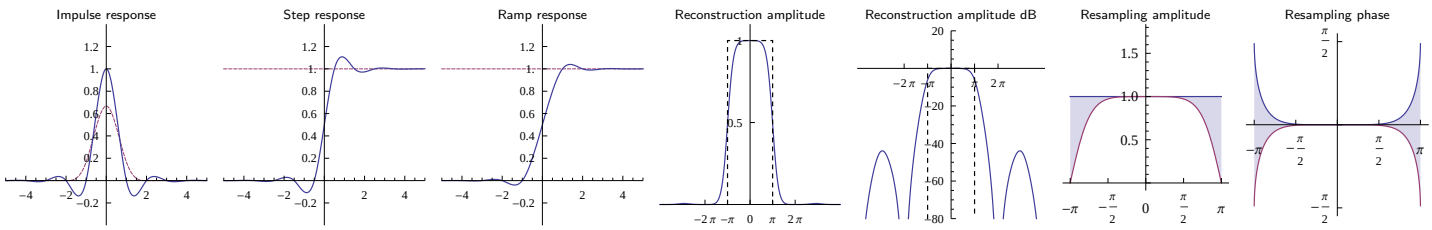
`somoms5`: SO-MOMS-5 of [Blu et al. 2001]



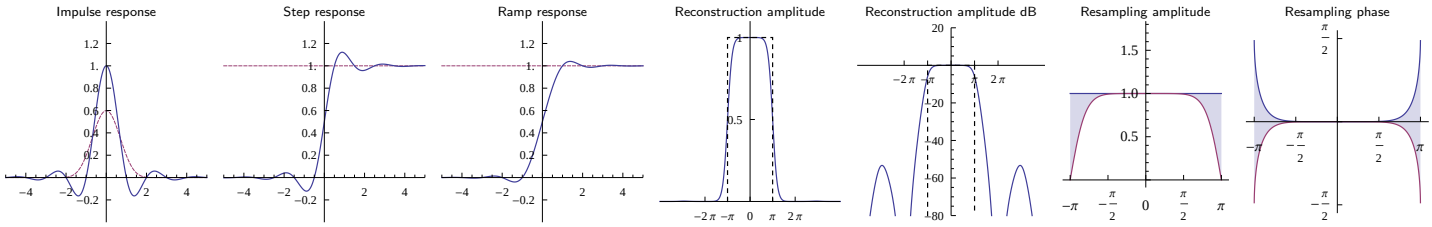
`bspline2i`: B-spline interpolation $(\beta^2)_{\text{int}}$



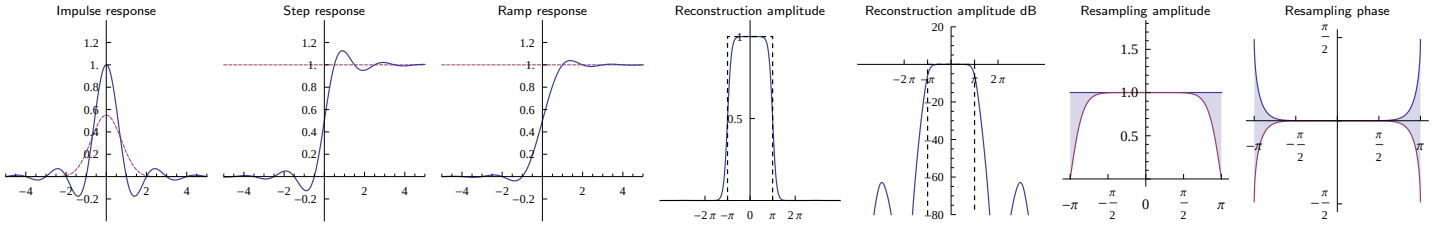
`bspline3i`: B-spline interpolation $(\beta^3)_{\text{int}}$



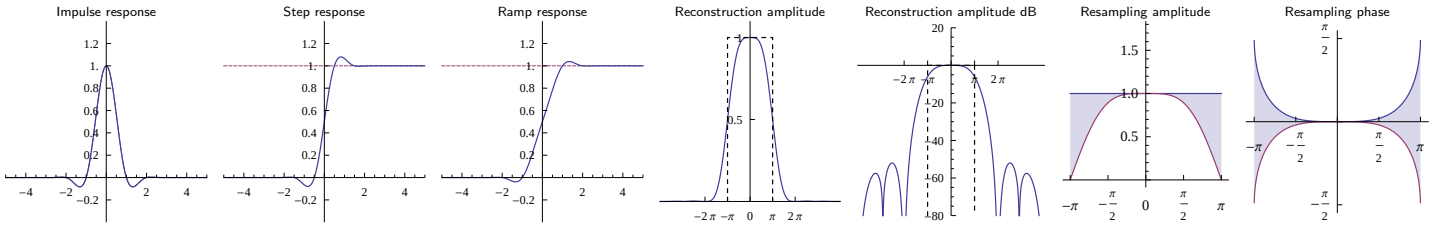
`bspline4i`: B-spline interpolation $(\beta^4)_{\text{int}}$



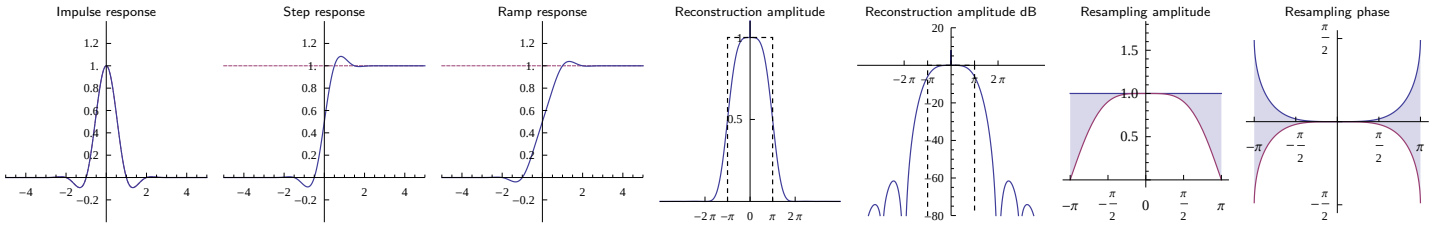
`bspline5i`: B-spline interpolation $(\beta^5)_{\text{int}}$



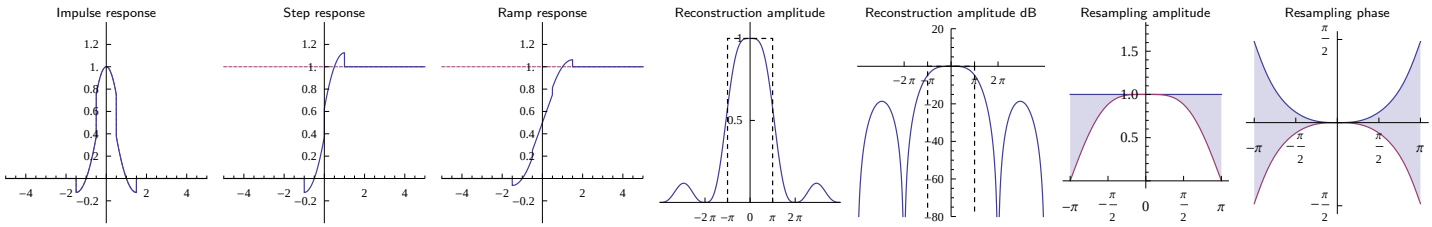
`meijering5`: 5^{th} degree interpolation [Meijering et al. 1999]



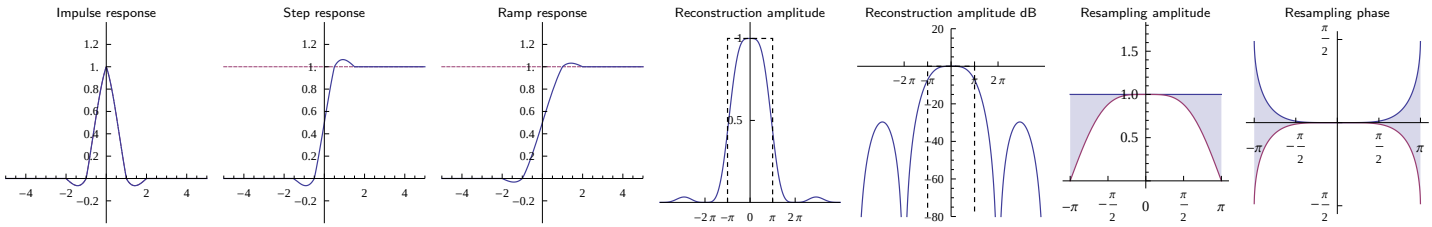
`meijering7`: 7^{th} degree interpolation [Meijering et al. 1999]



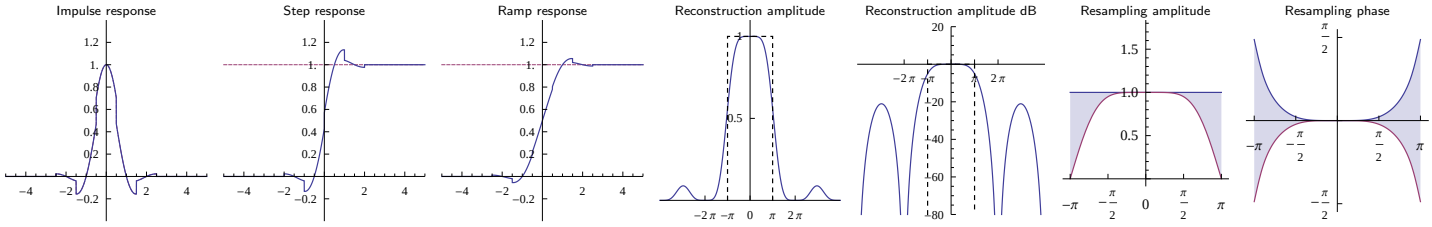
`schaum2`: 2^{th} degree local-Lagrangian interpolation Schaum [1993] (also I-MOMS-2 [Blu et al. 2001])



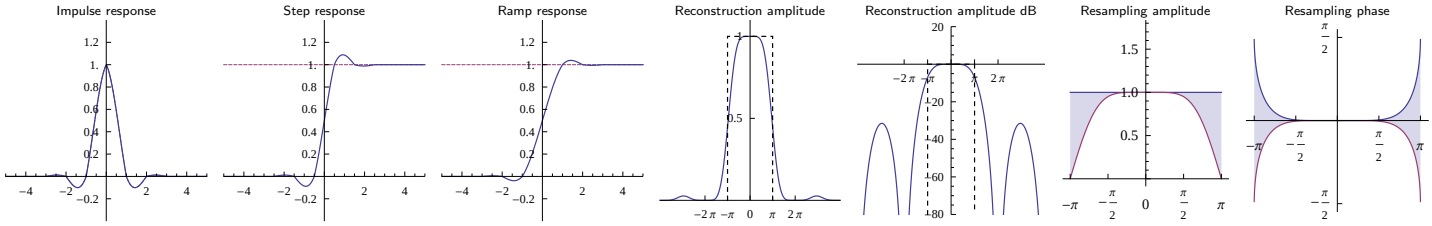
`schaum3`: 3^{th} degree local-Lagrangian interpolation Schaum [1993] (also4I-MOMS-3 [Blu et al. 2001])



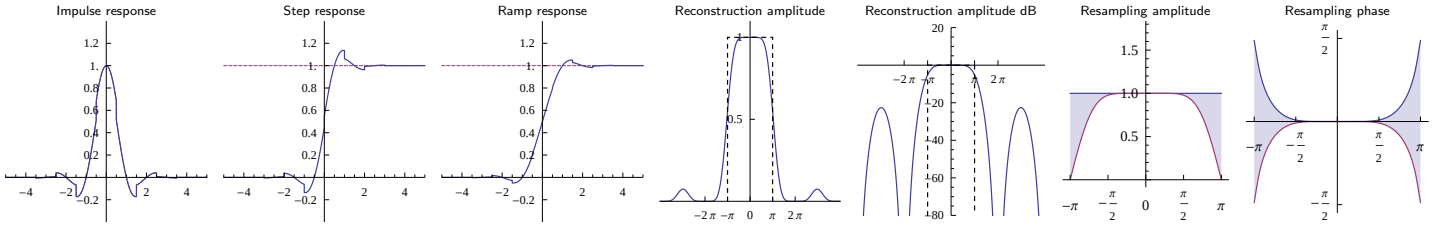
`schaum4`: 4^{th} degree local-Lagrangian interpolation Schaum [1993] (also6I-MOMS-4 [Blu et al. 2001])



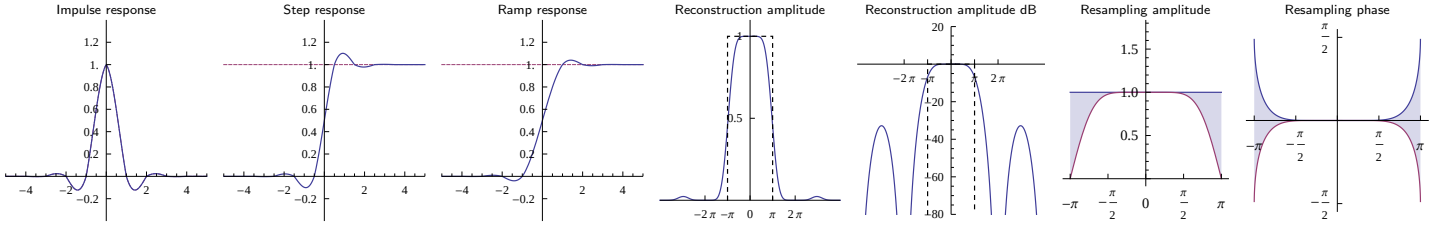
`schaum5`: 5^{th} degree local-Lagrangian interpolation Schaum [1993] (also I-MOMS-5 [Blu et al. 2001])



`schaum6`: 6^{th} degree local-Lagrangian interpolation Schaum [1993] (also I-MOMS-6 [Blu et al. 2001])

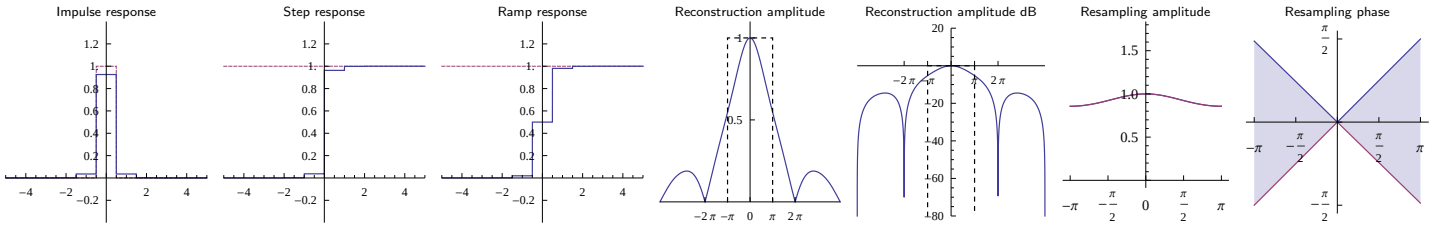


`schaum7`: 7^{th} degree local-Lagrangian interpolation Schaum [1993] (also I-MOMS-7 [Blu et al. 2001])

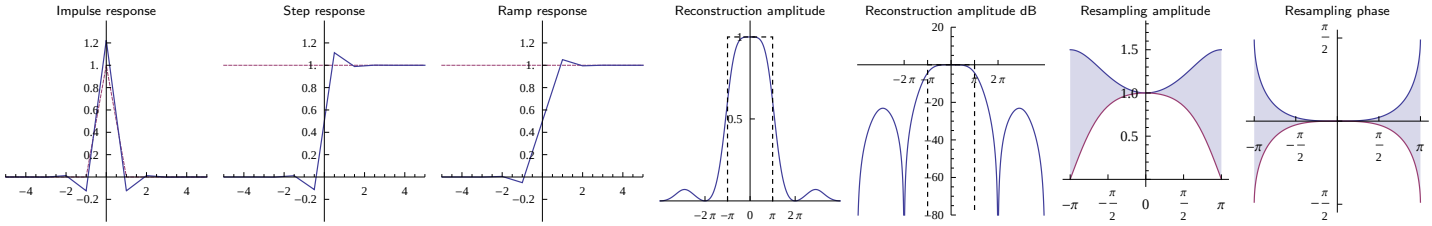


Approximating schemes

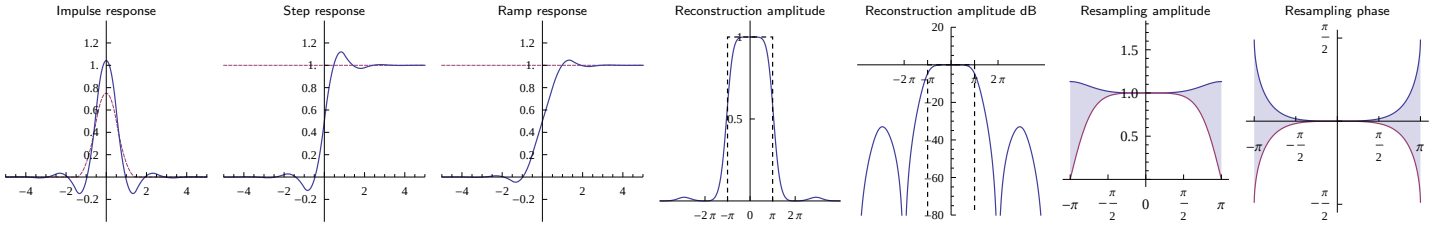
`condat0`: Quasi-interpolator of Condat et al. [2005] using β^0



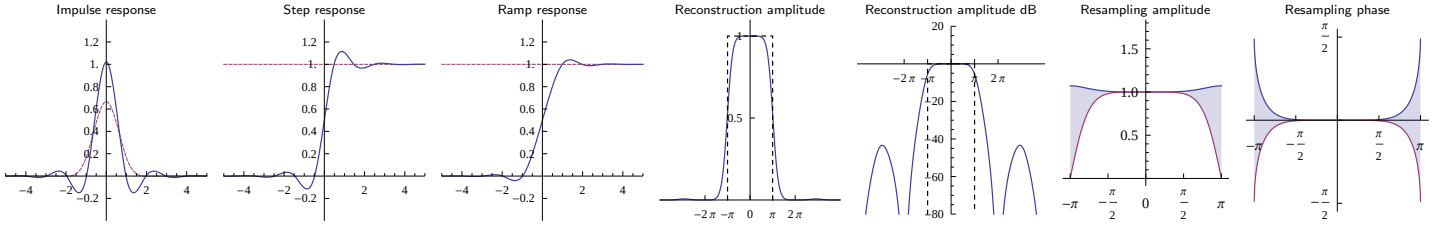
`condat1`: Quasi-interpolator of Condat et al. [2005] using β^1



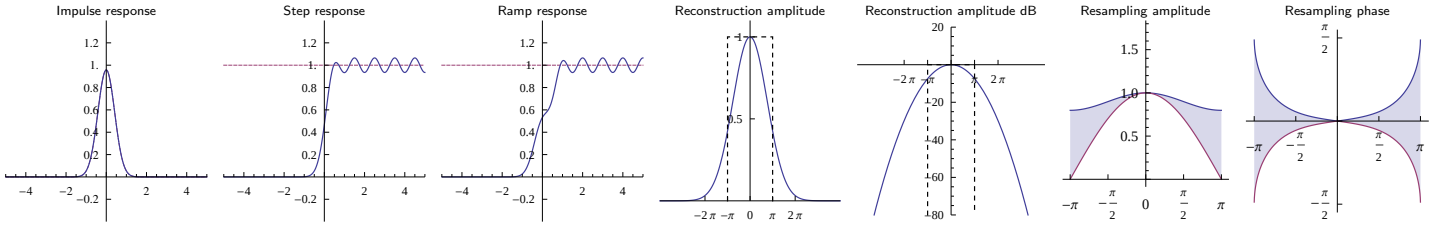
`condat2`: Quasi-interpolator of Condat et al. [2005] using β^2



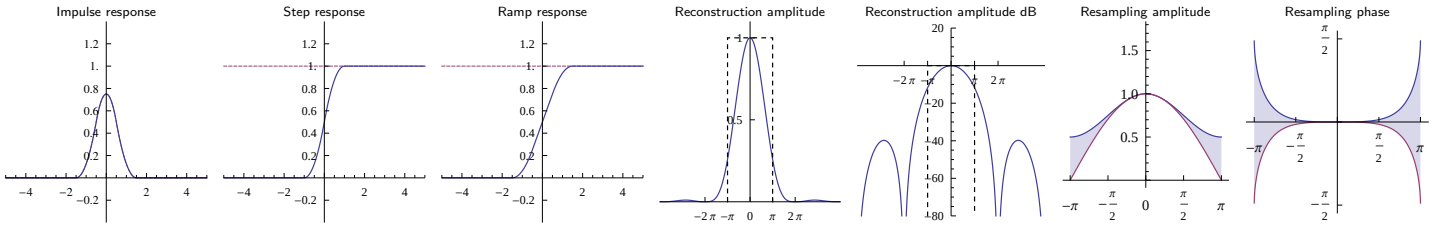
`condat3`: Quasi-interpolator of Condat et al. [2005] using β^3



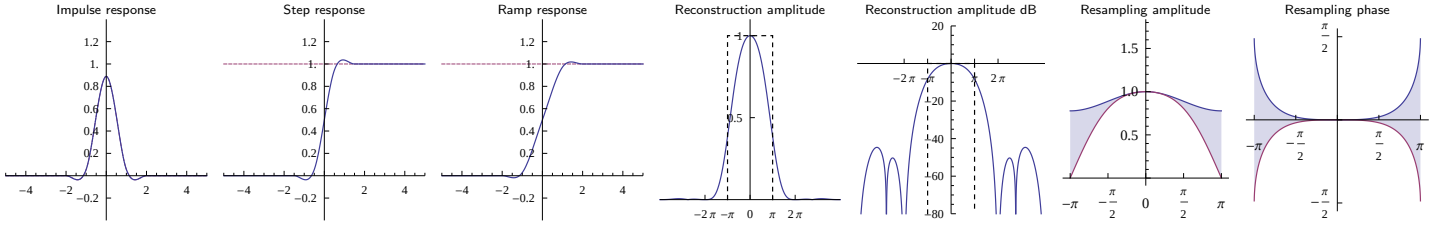
`gaussian2p5`: Gaussian with $\sigma = \frac{5}{12}$



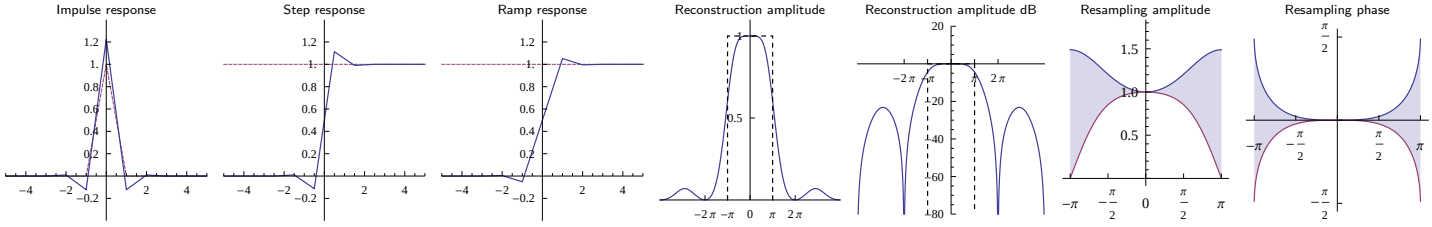
`adodgson`: Approximating quadratic of Dodgson [1997]



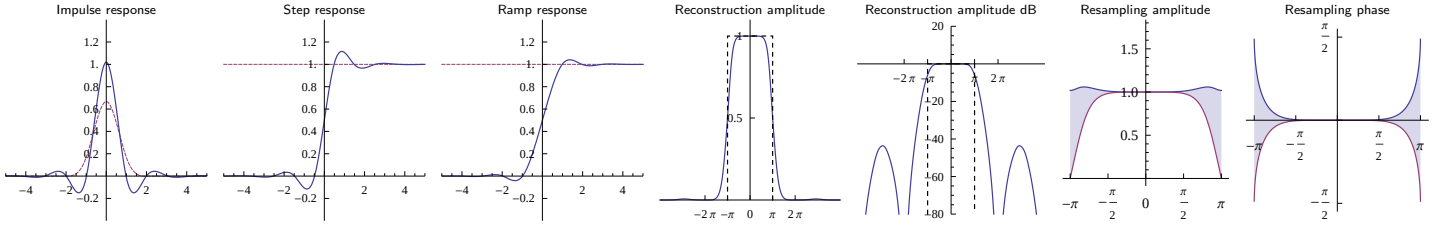
`mitchell`: [Mitchell and Netravali 1988] with $B = C = \frac{1}{3}$



`dalai1`: Quasi-interpolator of Dalai et al. [2005]

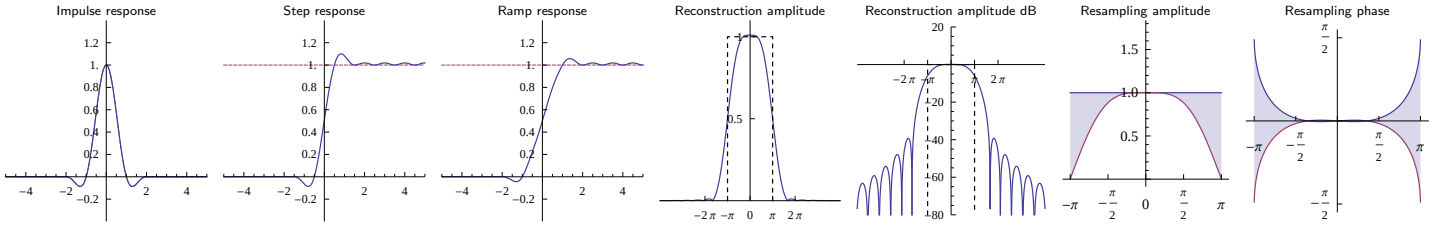


`quasiblu35`: Quasi-interpolator of Blu and Unser [1999] using β^3

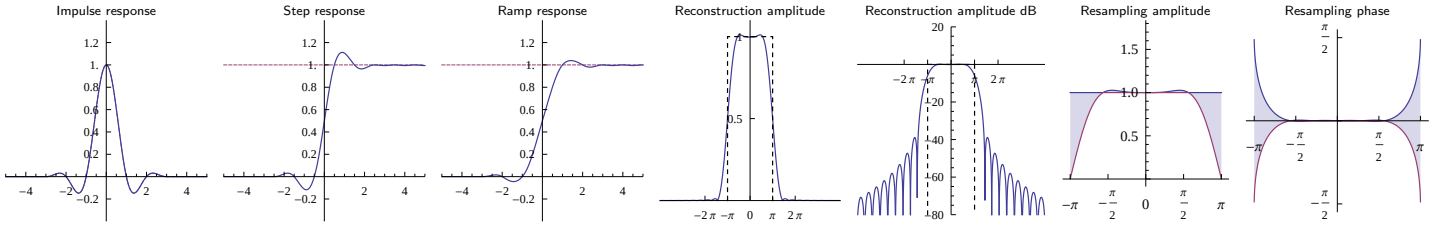


Windowed sinc **approximations**

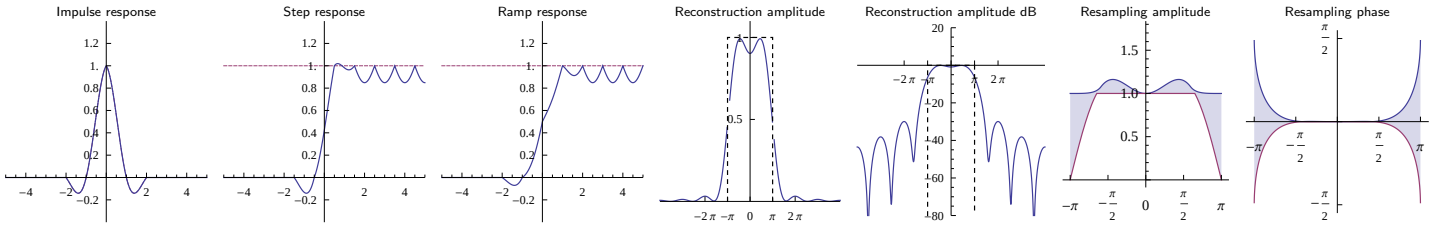
`lanczos4`: Lanczos window with $W = 4$



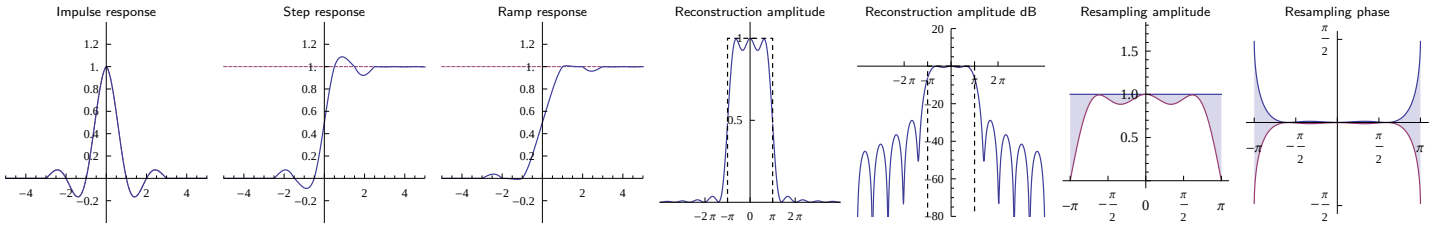
`lanczos6`: Lanczos window with $W = 6$



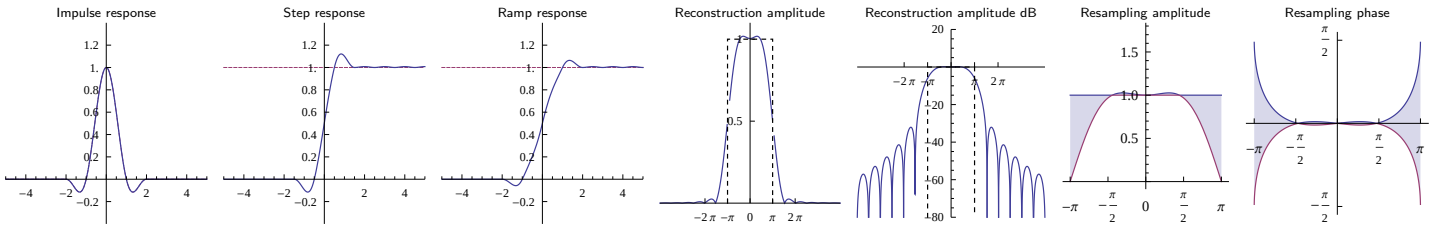
`bartlett4`: Bartlett window with $W = 4$



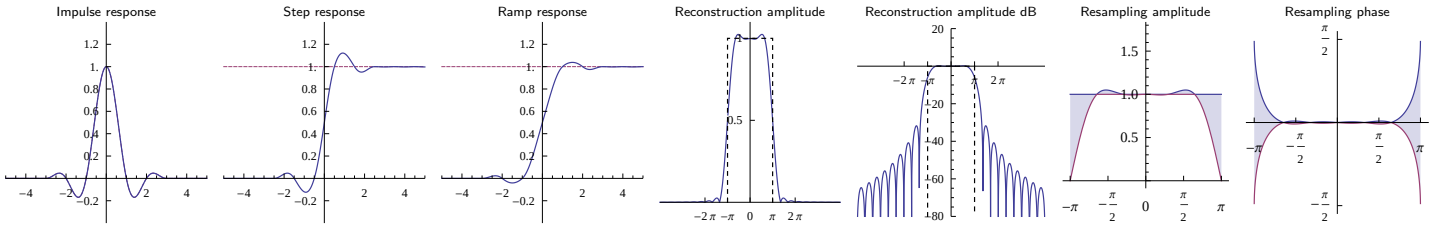
`bartlett6`: Bartlett window with $W = 6$



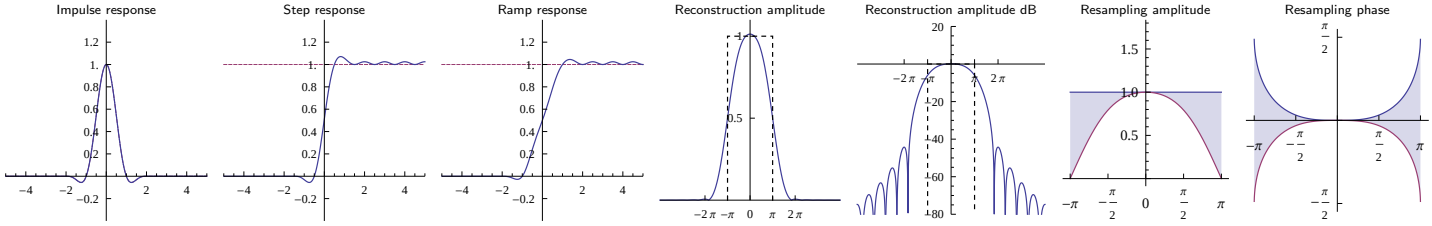
`welch4`: Welch window with $W = 4$



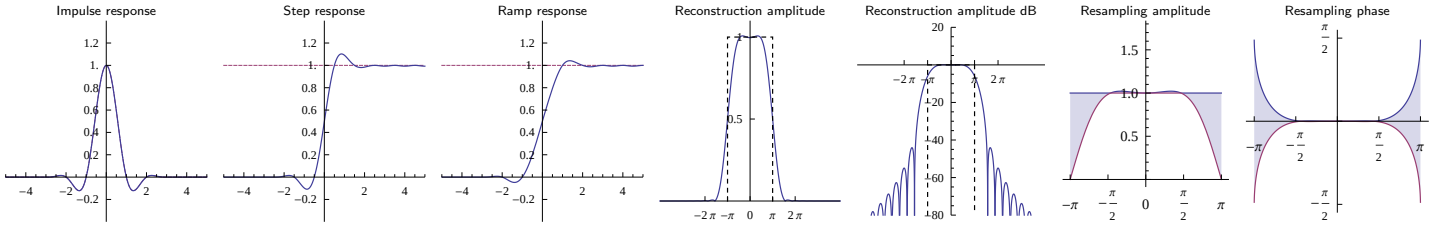
`welch6`: Welch window with $W = 6$



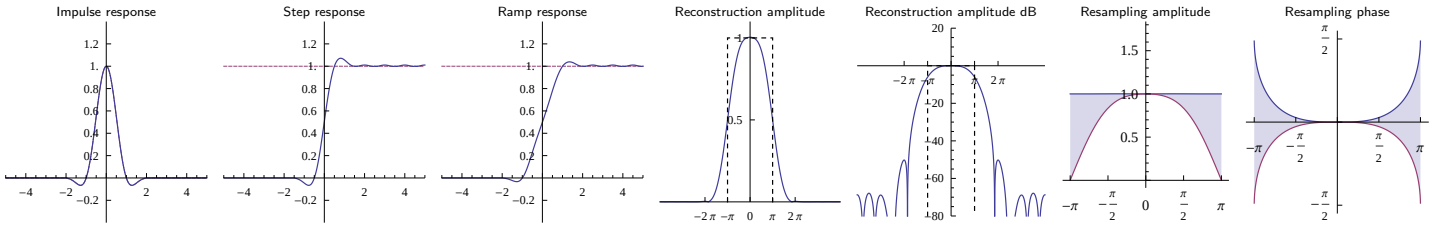
`hann4`: van Hann window with $W = 4$



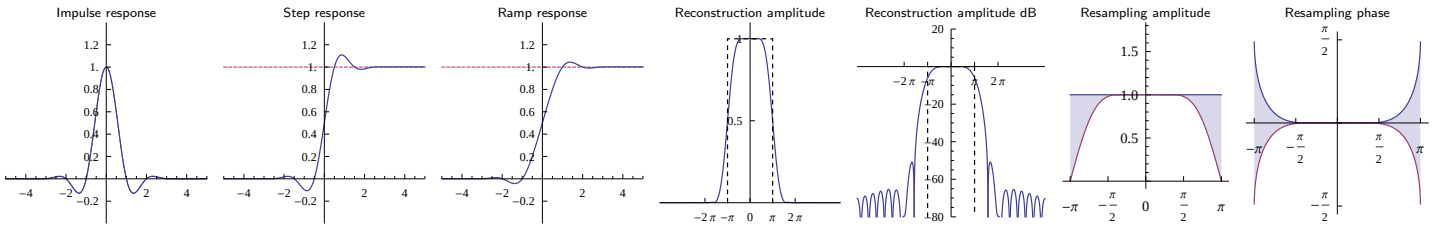
`hann6`: van Hann window with $W = 6$



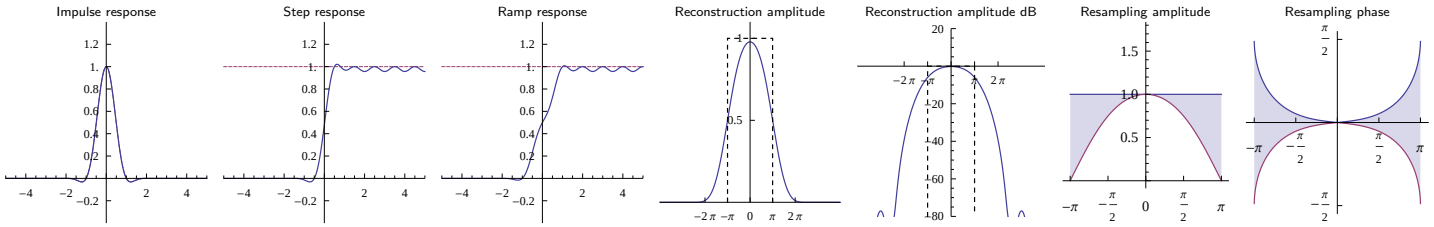
`hamming4`: Hamming window with $W = 4$



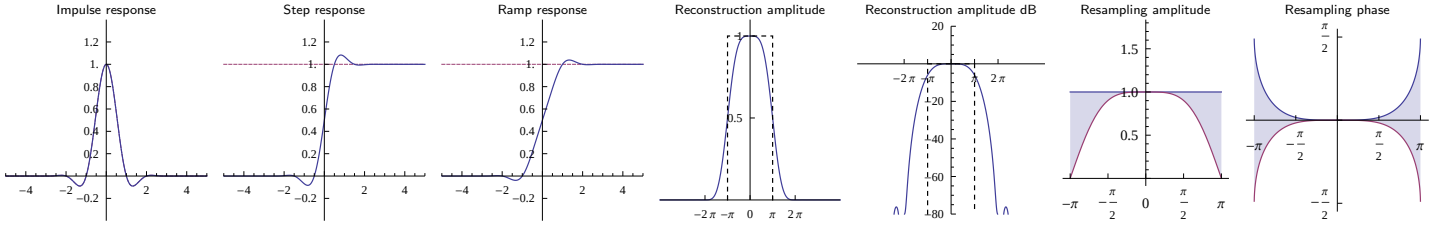
`hamming6`: Hamming window with $W = 6$



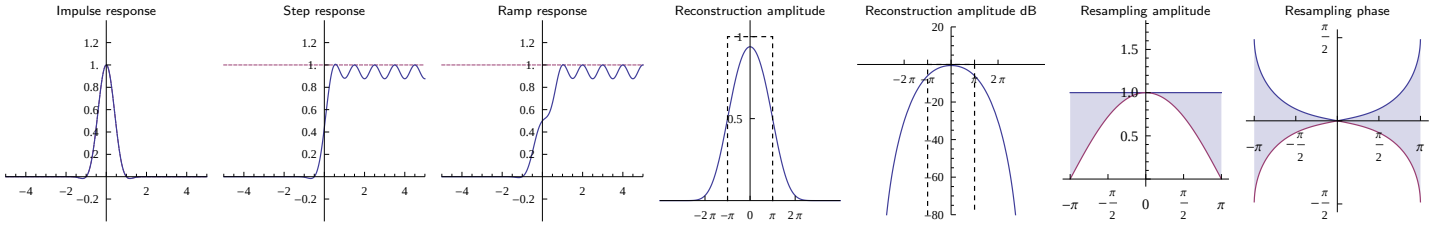
`blackman4`: Blackman window with $W = 4$



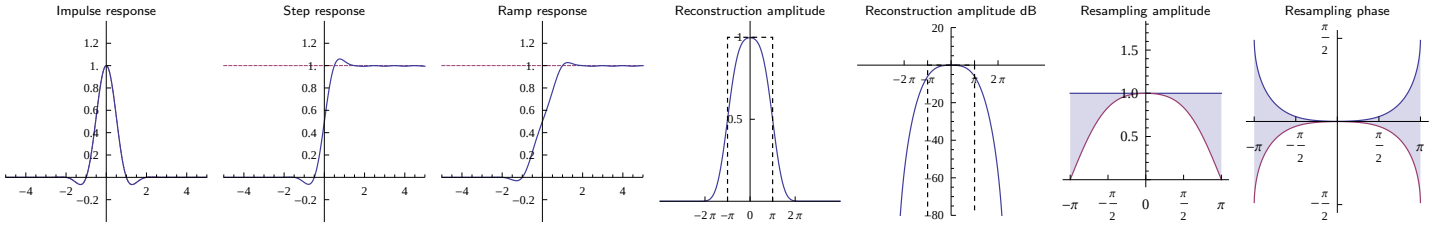
`blackman6`: Blackman window with $W = 6$



`blackmanharris4`: Blackman-Harris window with $W = 4$



`blackmanharris6`: Blackman-Harris window with $W = 6$



References

Blu, T., Thévenaz, P., and Unser, M. 2001. MOMS: Maximal-order interpolation of minimal support. *IEEE TIP*, 10 (7):1069–1080.

Blu, T., Thévenaz, P., and Unser, M. 2004. Linear interpolation revitalized. *IEEE TIP*, 13(5):710–719.

Blu, T. and Unser, M. 1999. Quantitative Fourier analysis of approximation techniques: Part I—Interpolators and projectors. *IEEE TSP*, 47(10):2783–2795.

Catmull, E. and Rom, R. 1974. A class of local interpolating splines. In *Computer Aided Geometric Design*, pages 317–326.

Condat, L., Blu, T., and Unser, M. 2005. Beyond interpolation: optimal reconstruction by quasi-interpolation. In *IEEE ICIP*, volume 1, pages 33–36.

Dalai, M., Leonardi, R., and Migliorati, P. 2005. Efficient digital pre-filtering for least-squares linear approximation. In *VLBV LNCS 3893/2006*, pages 161–169.

Dodgson, N. A. 1997. Quadratic interpolation for image resampling. *IEEE TIP*, 6(9):1322–1326.

Keys, R. G. 1981. Cubic convolution interpolation for digital image processing. *IEEE ASSP*, 29(6):1153–1160.

Meijering, E. H. W., Zuiderveld, K. J., and Viergever, M. A. 1999. Image reconstruction by convolution with symmetrical piecewise nth-order polynomial kernels. *IEEE TIP*, 8(2):192–201.

Mitchell, D. P. and Netravali, A. N. 1988. Reconstruction filters in computer graphics. *Computer Graphics (Proceedings of ACM SIGGRAPH 1988)*, 22(4):221–228.

Schaum, A. 1993. Theory and design of local interpolators. *CVGIP*, 55(6):464–481.