Automating variational inference for statistics and data mining

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A common situation

- You have a dataset
- Some models in mind
- Want to fit many different models to the data

Model-based psychometrics

$$y_{ij} \sim f(y \mid \alpha_i, \beta_j, \theta)$$

- Subjects i = 1,...,N
- Questions j = 1,...,J
- α_i = subject effect
- β_j = question effect
- θ = other parameters

The problem

- Inference code is difficult to write
- As a result:
  – Only a few models can be tried
  – Code runs too slow for real datasets
  – Only use models with available code
- How to get out of this dilemma?

Infer.NET: An inference compiler

- You specify a statistical model
- It produces efficient code to fit the model to data
- Multiple inference algorithms available:
  - Variational message passing
  - Expectation propagation
  - Gibbs sampling (coming soon)
- User extensible

Infer.NET: An inference compiler

- A compiler, not an application
- Model can be written in any .NET language (C++, C#, Python, Basic,…)
  - Can use data structures, functions of the parent language (jagged arrays, if statements, …)
- Generated inference code can be embedded in a larger program
- Freely available at:

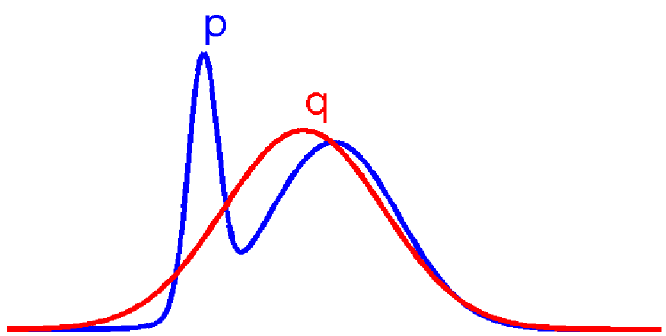
  http://research.microsoft.com/infernet

Papers using Infer.NET

- Benjamin Livshits, Aditya V. Nori, Sriram K. Rajamani, Anindya Banerjee, *“Merlin: Specification Inference for Explicit Information Flow Problems”,* Prog. Language Design and Implementation, 2009
- Vincent Y. F. Tan, John Winn, Angela Simpson, Adnan Custovic, *“Immune System Modeling with Infer.NET”,* IEEE International Conference on e-Science, 2008
- David Stern, Ralf Herbrich, Thore Graepel, *“Matchbox: Large Scale Online Bayesian Recommendations”,* WWW 2009
- Kuang Chen, Harr Chen, Neil Conway, Joseph M. Hellerstein, Tapan S. Parikh, *“Usher: Improving Data Quality With Dynamic Forms”,* ICTD 2009

Variational Bayesian inference

- True posterior is approximated by a simpler distribution (Gaussian, Gamma, Beta, …)
  - “Point-estimate plus uncertainty”
  - Halfway between maximum-likelihood and sampling



p=true
q=approx

Variational Bayesian inference

- Let variables be $x_1, \ldots, x_V$
- For each x_v pick an approximating family $q(x_v)$ (Gaussian, Gamma, Beta, …)
- Find the joint distribution $q(x) = \prod_v q(x_v)$

  that minimizes the divergence

  $$KL(q(x) \,||\, p(x \mid data))$$ (or other error measure)

Variational Bayesian inference

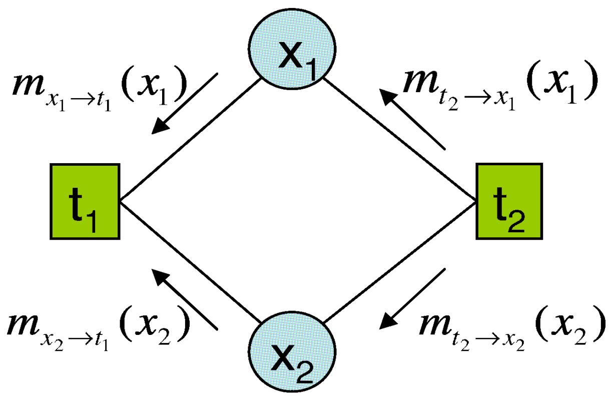
- Well-suited to large datasets, sequential processing (in style of Kalman filter)
- Provides Bayesian model score

Implementation

- Convert model into factor graph
- Pass messages on the graph until convergence

$$p(y \mid x) = p(y_1 \mid x_1, x_2)\, p(y_2 \mid x_1, x_2)$$

$\boxed{t_1}$ $\boxed{t_2}$



Further reading

- C. Bishop, *Pattern Recognition and Machine Learning*. Springer, 2006.
- T. Minka, “Divergence measures and message passing,” Microsoft Tech. Rep., 2005.
- T. Minka & J. Winn, “Gates,” NIPS 2008.
- M.J. Beal & Z. Ghahramani, “The Variational Bayesian EM Algorithm for Incomplete Data: with Application to Scoring Graphical Model Structures,” *Bayesian Statistics* 7, 2003.

Example: Cognitive Diagnosis Models (DINA,NIDA)

B. W. Junker and K. Sijtsma, "Cognitive Assessment Models with Few Assumptions, and Connections with Nonparametric Item Response Theory," *Applied Psychological Measurement* 25: 258-272 (2001)

- $y_{ij} = 1$ if student i answered question j correctly (observed)
- if question j requires skill k (known)
- $q_{jk} = 1$ if student i has skill k (latent)

$$hasSkill_{ik} = 1$$

$$hasSkill_{ik} \sim Bernoulli(pSkill_k)$$

- **DINA model**: K+2J parameters

$$hasSkills_{ij} = \prod_k hasSkill_{ik}^{q_{jk}}$$ (student possesses all skills for question)

$$p(y_{ij} = 1) = (1 - slip_j)^{hasSkills_{ij}} guess_j^{1 - hasSkills_{ij}}$$

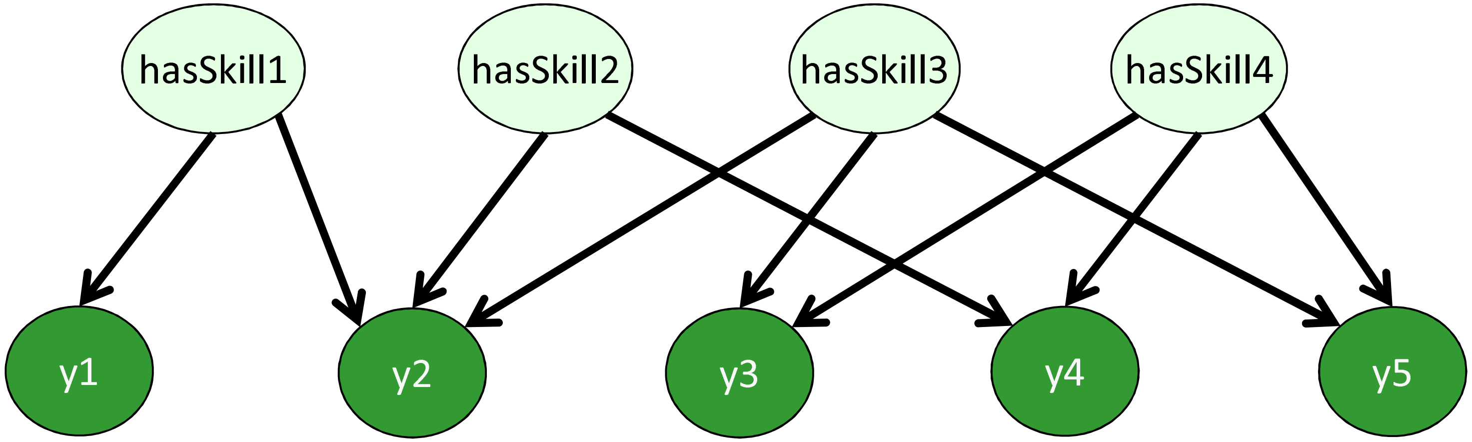
- **NIDA model**: K+2K parameters

$$exhibitsSkill_{ik} = (1 - slip_k)^{hasSkill_{ik}} guess_k^{1 - hasSkill_{ik}}$$

$$p(y_{ij} = 1) = \prod_k exhibitsSkill_{ik}^{q_{jk}}$$

Graphical model

(per student)



Linkage depends on the Q matrix

Prior work

- Junker & Sijtsma (2001), Anozie & Junker (2003) found that MCMC was effective but slow to converge
- Ayers, Nugent & Dean (2008) proposed clustering as fast alternative to DINA model
- What about variational inference?

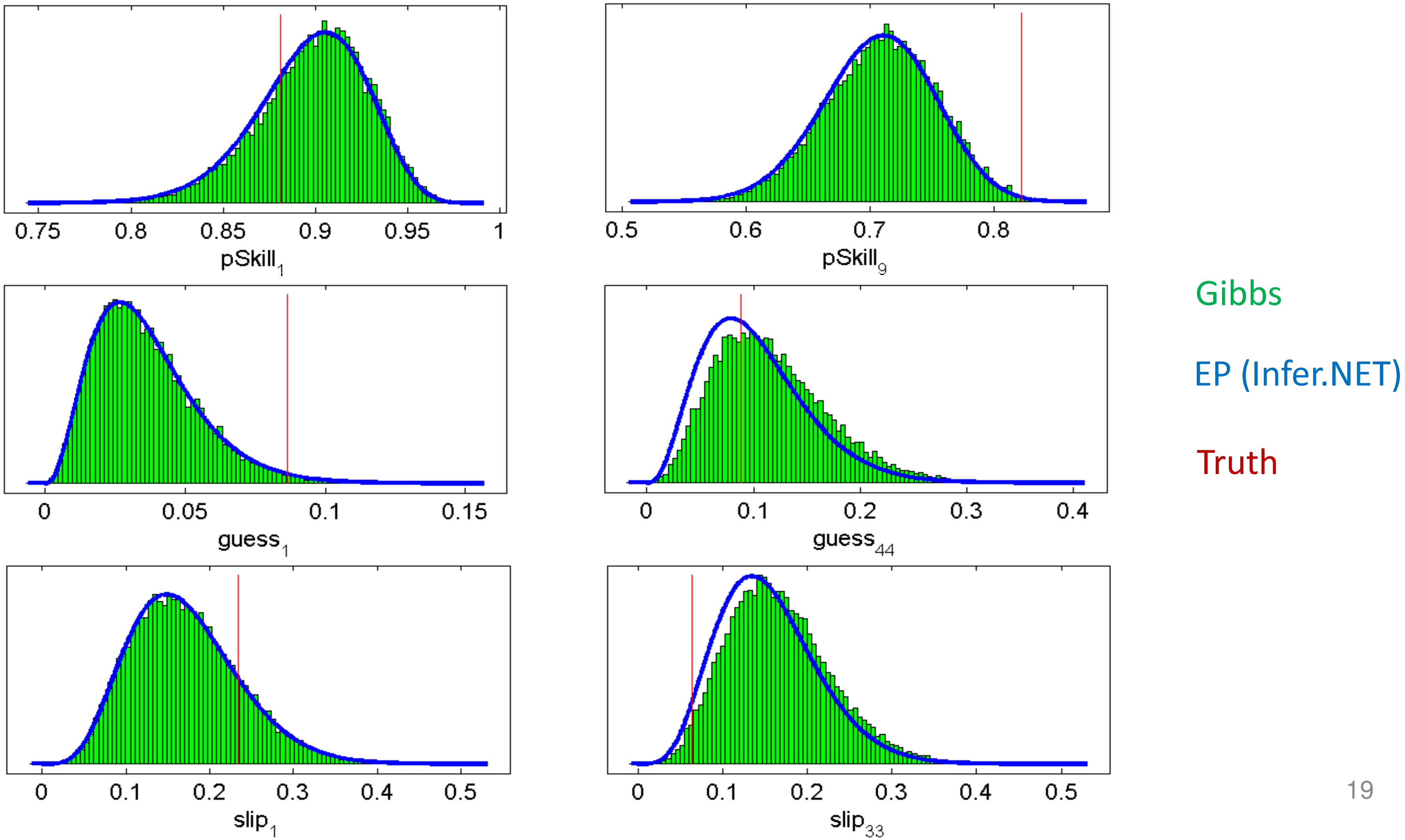
DINA,NIDA models in Infer.NET

- Each model is approx 50 lines of code
- Tested on synthetic data generated from the models
  - 100 students, 100 questions, 10 skills
  - Random question-skill matrix
  - Each question required at least 2 skills
- Infer.NET used Expectation Propagation (EP) with Beta distributions for parameter posteriors
  - Variational Message Passing gave similar results on DINA, couldn't be applied to NIDA

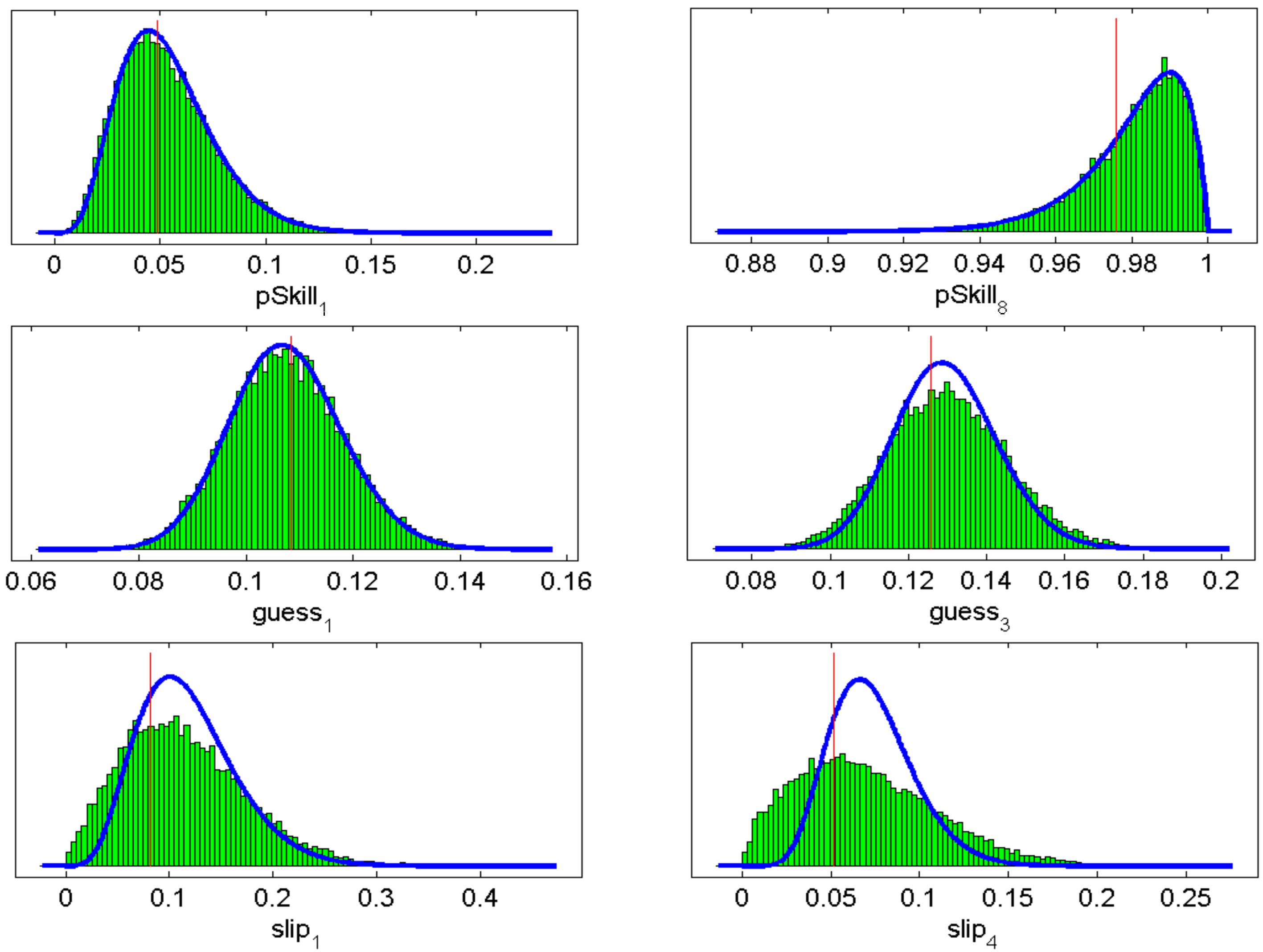
Comparison to BUGS

- EP results compared to 20,000 samples from BUGS
- For estimating posterior means, EP is as accurate as 10,000 samples, for same cost as 100 samples
  - i.e. 100x faster

DINA model on DINA data



NIDA model on NIDA data

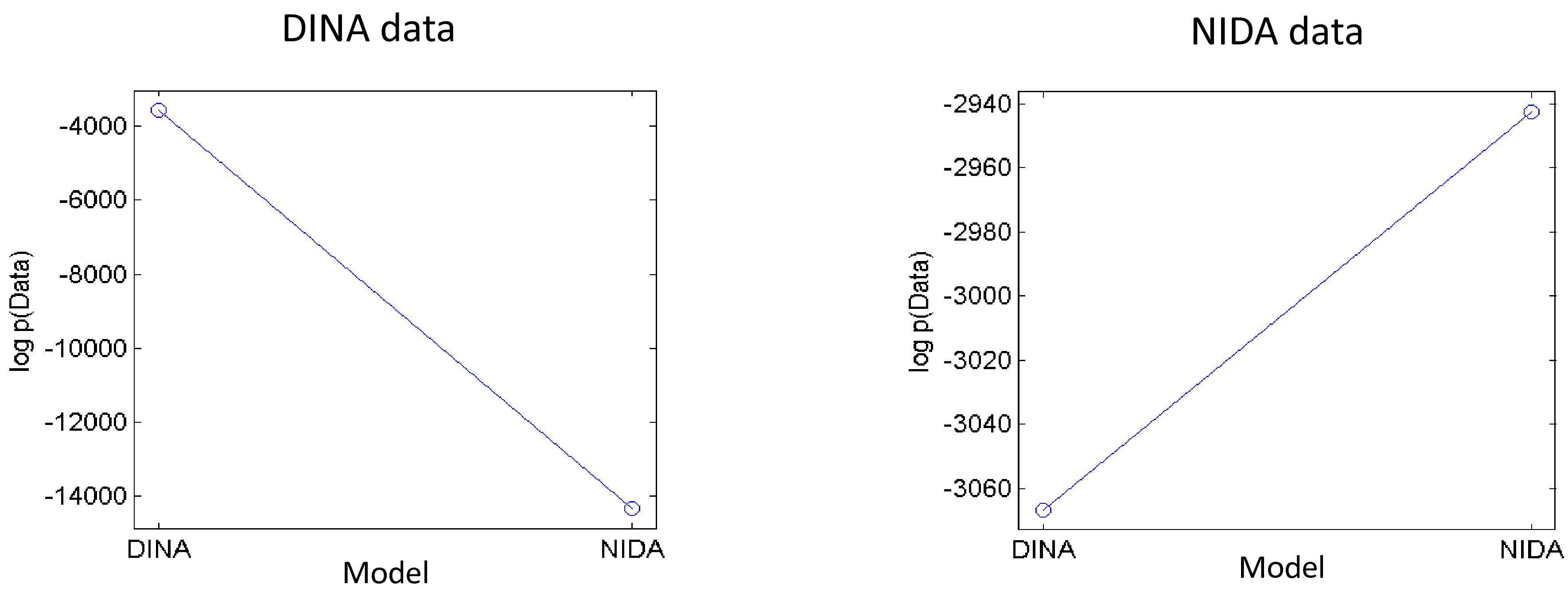


Gibbs

EP (Infer.NET)

Truth

Model selection



- Correct generative model is chosen in each case
- DINA model is better at fitting NIDA data than vice versa

Code for DINA model

```
using (Variable.ForEach(student)) {
  using (Variable.ForEach(question)) {
    VariableArray<bool> hasSkills =
      Variable.Subarray(hasSkill[student], skillsRequiredForQuestion[question]);
    Variable<bool> hasAllSkills = Variable.AllTrue(hasSkills);
    using (Variable.If(hasAllSkills)) {
      responses[student][question] = !Variable.Bernoulli(slip[question]);
    }
    using (Variable.IfNot(hasAllSkills)) {
      responses[student][question] = Variable.Bernoulli(guess[question]);
    }
  }
}
```

Code for NIDA model

```
using (Variable.ForEach(skillForQuestion)) {
  using (Variable.If(hasSkills[skillForQuestion])) {
    showsSkill[skillForQuestion] = !Variable.Bernoulli(slipSkill[skillForQuestion]);
  }
  using (Variable.IfNot(hasSkills[skillForQuestion])) {
    showsSkill[skillForQuestion] = Variable.Bernoulli(guessSkill[skillForQuestion]);
  }
}
responses[student][question] = Variable.AllTrue(showsSkill);
```

Example: Latent class models for diary data

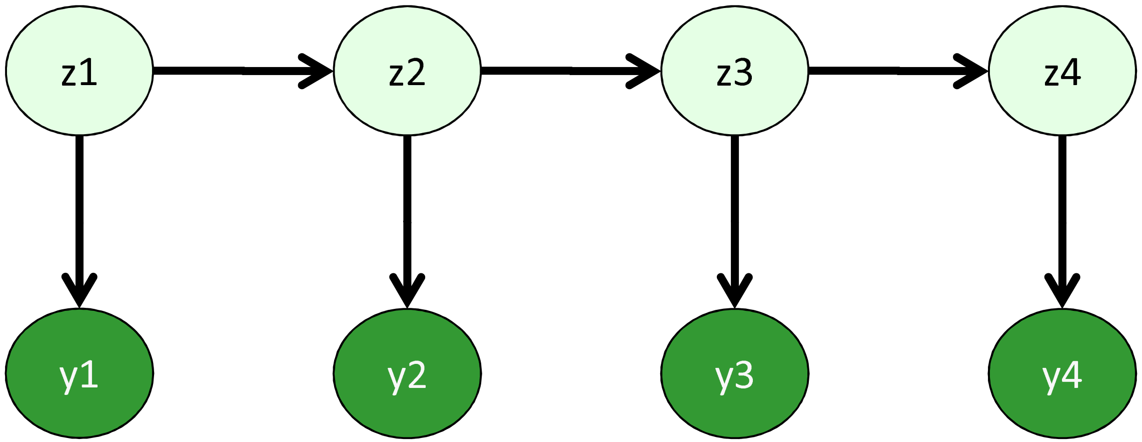
F. Rijmen and K. Vansteelandt and P. De Boeck, “Latent class models for diary method data: parameter estimation by local computations,” *Psychometrika*, 73, 167-182 (2008)

Diary data

- Patients assess their emotional state over time (Rijmen et al 2008, PMKA)
- $y_{itj} = 1$ if subject i at time t feels emotion j (observed)

Basic Hidden Markov model:

- $z_{it} \in \{1, \ldots, S\}$ is hidden state of subject i at time t (latent)



S^2 transition parameters

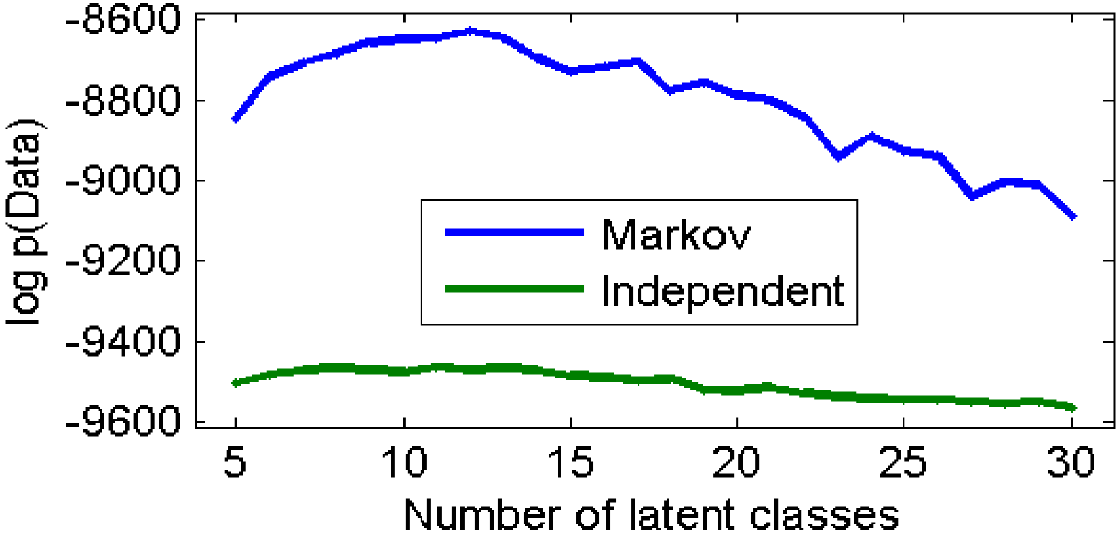
JS observation parameters

Prior work

- Rijmen et al (2008) used maximum-likelihood estimation of HMM parameters
  - model selection was an open issue
- Which model gets highest score from variational Bayes?

HMM in Infer.NET

- Model is approx 70 lines of code
- Can vary:
  – number of latent classes (S)
  – whether states are independent or Markov



Best model is Markov
with 12 latent classes

Hierarchical HMM

- Real data has more structure than HMM
- 32 subjects were observed over 7 days, having 9 observations per day
  - Basic HMM treated each day independently
- Rijmen et al (2008) proposed switching between different HMMs on different days (hierarchical HMM)
  - more model selection issues

Hierarchical HMM in Infer.NET

- Model is approx 100 lines of code
- Can additionally vary:
  - number of HMMs (1,3,5,7,9)
  - whether days are independent or Markov
  - whether transition params depend on day
  - whether observation params depend on day
- Best model among 400 combinations (2 hours using VMP):
  - 5 HMMs, each having 5 latent states
  - Observation params depend on day, but transition params do not

Summary

- Infer.NET allowed 4 custom models to be implemented in a short amount of time
- Resulting code was efficient enough to process large datasets, compare many models
- Variational inference is potential replacement for sampling in DINA,NIDA models

http://research.microsoft.com/infernet

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